Homework Assignment #1 – due via Moodle at 11:59 pm on Monday, Sept. 15, 2025 [Graded Prob. 2 edited 9/10/25]

Instructions, notes, and hints:

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

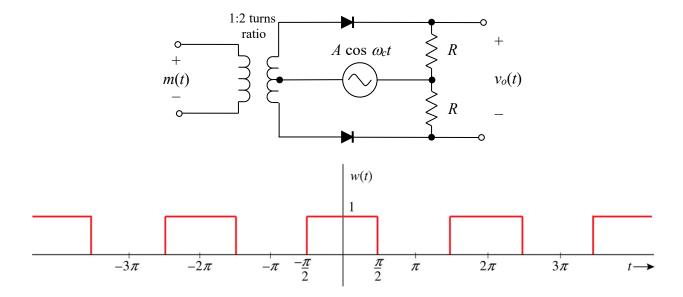
The first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

Graded Problems:

1. [Adapted from Prob. 4.2-5 of Lathi & Ding, 4th ed.] Amplitude modulators and demodulators can be built without using multipliers. In the upper figure below, m(t) is a message (baseband) signal, and $A \cos \omega_c t$ is a local oscillator at the carrier frequency f_c . The local oscillator has a large amplitude such that A >> |m(t)|. The two diodes are identical and have an equivalent resistance r when conducting (forward biased) and infinite resistance when reverse biased. Show that the output voltage $v_o(t)$ is given by

$$v_o(t) = \frac{2R}{R+r} w(t) m(t),$$

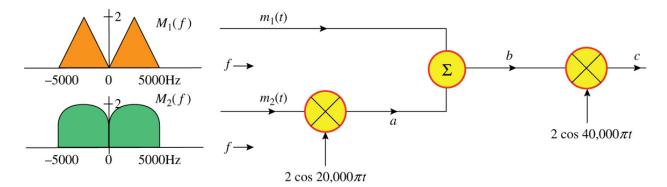
where w(t) is the switching periodic signal shown below with period $2\pi/\omega_c$ seconds (not 2π as indicated in the figure).



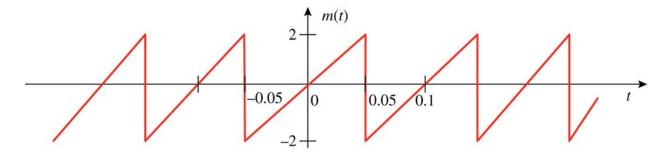
2. [boldface text added and w(t) edited 9/10/25] Show that the circuit in the previous problem can be used as a DSB-SC modulator if it is followed by a bandpass filter centered at the carrier frequency f_c . The Fourier series of the switching waveform w(t) is given below; the even n terms are zero.

$$w(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin\left(\frac{n\pi}{2}\right) \cos\left(n\omega_c t\right)$$

- **3.** [Adapted from Prob. 4.2-7 of Lathi & Ding, 6^{th} ed.] Two signals $m_1(t)$ and $m_2(t)$, both band-limited to 5000 Hz, are to be transmitted simultaneously over a channel by the multiplexing scheme shown below. The signal at point b is the multiplexed signal, which modulates a carrier of frequency 20 kHz. The modulated signal at point c is transmitted over a channel.
 - **a.** Sketch signal spectra at points a, b, and c.
 - **b.** Find the bandwidth of the signal at point c.



- **4.** [Adapted from Prob. 4.3-2 of Lathi & Ding, 4^{th} ed.] Sketch the AM signal $[A + m(t)] \cos \omega_c t$ for the periodic sawtooth signal m(t) shown below corresponding to each modulation index given below, where A is the amplitude of the carrier signal. Assume that the bandwidth B of the message signal m(t) satisfies $B \ll f_c$.
 - **a.** $\mu = 0.5$
 - **b.** $\mu = 1$
 - **c.** $\mu = 2$
 - **d.** $\mu = \infty$. In this case, what type of modulation is the signal equivalent to?



Ungraded Problem:

1. [Adapted from Prob. 4.3-8 of Lathi & Ding, 4^{th} ed.] AM signals can be demodulated by a crystal detector followed by a low-pass filter and a DC blocker (a capacitor) as shown below, even if the incoming AM signal is not amplified. This was the usual case for so-called crystal radios, which were widely used in the early days of radio because they were inexpensive and relatively easy to assemble by nonprofessionals. The crystal detector was often made from a thin wire called a "cat's whisker" that probed the surface of a galena crystal; it operated much like a modern semiconductor diode. Assume that a crystal detector can be approximated as a squaring device, that is, that the output is proportional to the square of the input. (It is nonlinear.) Find mathematical representations of the signals at points a, b, and c if the AM signal has the form given below, where A and ω_c are the amplitude and radian frequency, respectively, of the carrier and m(t) is the message (baseband) signal. Assume that the bandwidth of the message signal is much smaller than f_c . Indicate the distortion term in the output y(t), and show that if A >> |m(t)|, the distortion is small.

$$S_{AM}(t) = [A + m(t)] \cos \omega_c t$$

$$S_{AM}(t) \circ \underbrace{ ()^2 \quad a \quad \text{LPF} \quad b \quad \text{DC block} \quad c \circ y(t)}_{}$$

References:

The text "Lathi & Ding, 4th ed." refers to B. P. Lathi and Zhi Ding, *Modern Digital and Analog Communication Systems*, 4th ed., Oxford University Press, 2009.

The waveforms shown in Graded Probs. 1 and 4 and the block diagram shown in Graded Prob. 3 are from the ECEG 470/670 course textbook (B. P. Lathi and Zhi Ding, *Modern Digital and Analog Communication Systems*, 6th ed., Oxford University Press, 2025).