ECEG 470/670 Communication and Information Systems Fall 2025

Homework Assignment #2 – due via Moodle at 11:59 pm on Friday, Sept. 26, 2025

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

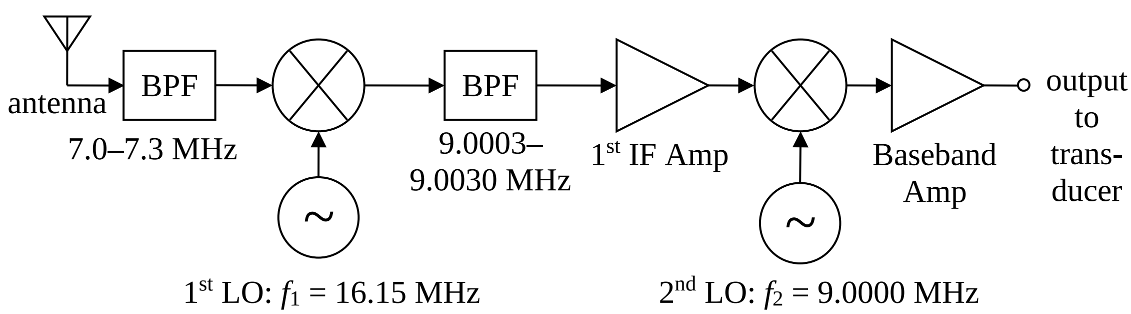
The first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. The diagram below depicts a typical receiver architecture for detecting single sideband signals. Depending on the setting of the second local oscillator's (LO) frequency, either USB or LSB can be demodulated.

   a. Find the carrier frequency of the signal being detected, and determine which sideband (USB or LSB) the receiver is configured to demodulate. Also determine whether the baseband signal's spectrum is inverted. That is, determine whether the low-frequency components of the baseband signal are shifted to the upper end of its spectrum and vice versa.

   b. Determine the new frequency setting required for f_2 to demodulate the opposite sideband of signals in the 7.0–7.3 MHz range.



2. [Adapted from Prob. 5.1-4 of Lathi & Ding, 4th ed.] Over an interval $|t| \le 1$, an angle-modulated signal is given by the expression shown below. It is known that the carrier frequency is $\omega_c = 10{,}000\pi t$. All time quantities are given in seconds and all radian frequencies are given in rad/s.

$$\varphi_{FM}(t) \quad \text{or} \quad \varphi_{PM}(t) = 10\cos(13{,}000\pi t + 0.2\pi)$$

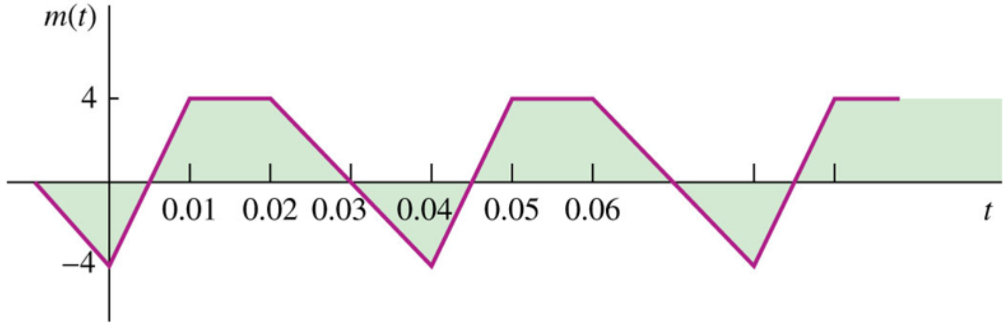
   a. Assuming a PM signal with $k_p = 1000$, determine $m(t)$ over the interval $|t| \le 1$.

   b. Assuming an FM signal with $k_f = 1000$, determine $m(t)$ over the interval $|t| \le 1$.

(*continued on next page*)

3. [Adapted from Prob. 5.2-2 of Lathi & Ding, 4th ed.] Assume that the bandwidth of the modulated signal $m(t)$ shown below is equal to its fifth harmonic frequency. The unit is seconds on the time axis.

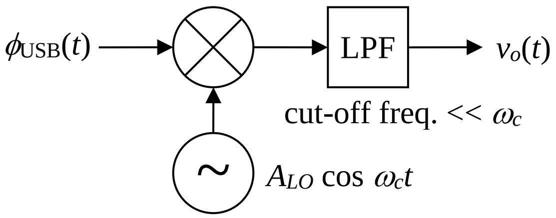
- **a.** Determine the approximate bandwidth of the FM signal.
- **b.** Determine the approximate bandwidth of the PM signal.



4. A tone-modulated USB signal experiences Doppler and phase shifts so that it can be expressed as

$$\phi_{\text{USB}}(t) = A_{RF}\{\cos[(\omega_c + \Delta\omega)t + \theta]\cos\omega_m t - \sin[(\omega_c + \Delta\omega)t + \theta]\sin\omega_m t\}$$

where $\Delta\omega$ accounts for the Doppler shift, θ is the phase shift relative to the local oscillator, and ω_m is the angular frequency of the modulating tone. Note that $\sin\omega_m t$ is the Hilbert transform of $\cos\omega_m t$. As shown in the block diagram below, a demodulator using the filter method has been designed to extract the original tone signal from the USB signal. The local oscillator generates the signal $A_{LO}\cos\omega_c t$ and sets the reference phase. Find an expression for the USB demodulator's output voltage $v_o(t)$ in the presence of $\Delta\omega$ and θ. *Hint*: The output voltage should be a frequency and phase-shifted version of the message (tone) signal $m(t) = \cos\omega_m t$.



***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

(*continued on next page*)

1. [Adapted from Prob. 4.4-7 of Lathi & Ding, 4th ed.] Suppose that a USB signal has been generated using the phasing method. See Fig. 4.11 in the textbook by Lathi & Ding, 6th ed. Determine whether the output of the modulator is still an SSB signal (LSB or USB) if the input is $m_h(t)$ instead of $m(t)$, that is, the Hilbert transform of the message signal, and whether the bandwidth is equal to that of $m(t)$. Also determine whether (and how) a demodulator can be devised to extract the message signal $m(t)$ from the transmitted signal.

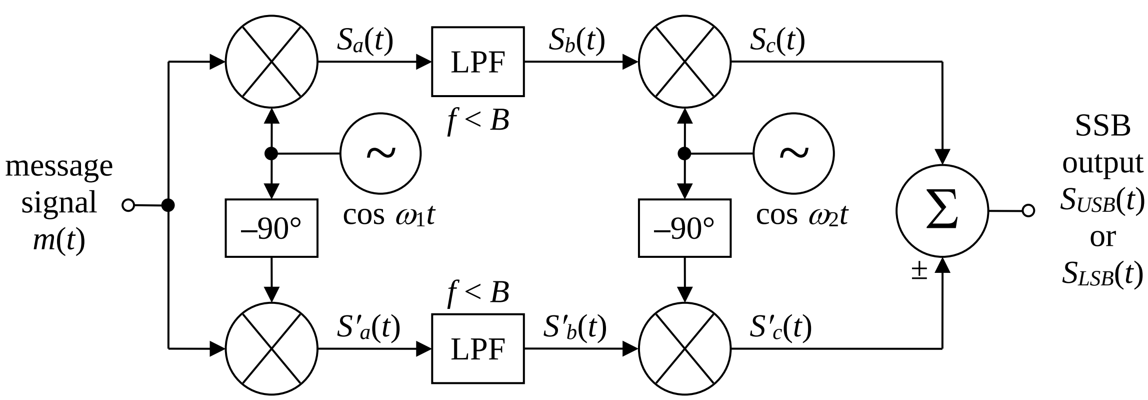
2. As explained in the textbook, two widely used methods for generating single sideband (SSB) signals are the filter method and the phasing method. A third method, called the Weaver method, is mentioned only briefly. The figure on the next page shows how the Weaver method is implemented. The low-pass filter labeled $f < B$ has a cut-off frequency equal to the bandwidth limit B (in Hz) of $m(t)$. The first oscillator frequency f_1 is also equal to B. The second oscillator frequency f_2 is typically much greater than f_1. A key advantage of the method is that it does not require the wide baseband signal to be phase-shifted as in the phasing method; another is that it does not require difficult-to-build highly selective filters with tiny percentage bandwidths as in the filter method. The Weaver method is therefore especially well-suited for generating SSB signals in the VHF range and above.

Suppose that the applied message signal has the form $m(t) = A\cos\omega_m t$, where ω_m is a relatively low baseband frequency and is less than or equal to some bandwidth B. The various labeled signals in the upper signal path would have the following time-domain forms:

$$S_a(t) = \frac{A}{2}\left[\cos(\omega_m + \omega_1)t + \cos(\omega_m - \omega_1)t\right] \qquad S_b(t) = \frac{A}{2}\cos(\omega_m - \omega_1)t$$

$$S_c(t) = \frac{A}{2}\cos(\omega_m - \omega_1)t\cos\omega_2 t = \frac{A}{4}\left[\cos(\omega_m - \omega_1 + \omega_2)t + \cos(\omega_m - \omega_1 - \omega_2)t\right]$$

- **a.** Find the expressions for the time-domain signals $S'_a(t)$, $S'_b(t)$ and $S'_c(t)$.
- **b.** Determine which sideband (USB or LSB) is associated with adding $S_c(t)$ and $S'_c(t)$ together, and which is associated with subtracting them.
- **c.** Suppose that a message signal has a bandwidth $B = 6$ kHz. Find the required frequencies f_1 and f_2 of the two oscillators and the operation type (addition or subtraction) that the summing junction must perform to generate a USB signal with a carrier frequency of 144.200 MHz and an upper limit of 144.206 MHz.



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3. [adapted from Prob. 5.2-7 of Lathi & Ding, 4th ed.] Given $m(t) = \sin 2000\pi t$, $k_f = 200{,}000\pi$, and $k_p = 10$,

- **a.** Estimate the bandwidths of $\phi_{FM}(t)$ and $\phi_{PM}(t)$, the FM and PM signals, respectively, that would be generated using the parameters given in the problem statement.
- **b.** Repeat part **a** if the message signal amplitude is doubled.
- **c.** Repeat part **a** if the message signal frequency is doubled.
- **d.** Comment on the sensitivity of FM and PM bandwidths to the spectrum of $m(t)$. Include what effect, if any, the frequency content has on the results.

References:

The text "Lathi & Ding, 4th ed." refers to B. P. Lathi and Zhi Ding, *Modern Digital and Analog Communication Systems*, 4th ed., Oxford University Press, 2009.

The waveform shown in Graded Prob. 3 is Fig. P4.5-5 from the ECEG 470/670 course textbook (B. P. Lathi and Zhi Ding, *Modern Digital and Analog Communication Systems*, 6th ed., Oxford University Press, 2025).