ECEG 470/670 Communication and Information Systems Fall 2025

Homework Assignment #3 – due via Moodle at 11:59 pm on Monday, Oct. 20, 2025
[Prob. 2 revised 10/17/25]

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

The first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. Suppose that an FM receiver is tuned to a signal represented by the expression below.

$$\phi_{FM}(t) = 50\cos\left[180.2\pi\times10^{6}t + 3.2\sin\left(3{,}000\pi t - 0.4\pi\right) + 8.8\cos\left(120\pi t\right)\right]$$

- **a.** Find the bandwidth of the FM signal. Remember that you need to find the maximum possible shift away from the carrier frequency.
- **b.** The signal is applied to an FM detector consisting of a differentiator followed by an envelope detector. Find the output of the detector.
- **c.** Assuming that any DC or high-frequency output products are eliminated by a blocking capacitor and low-pass filter, find the message signal $m(t)$ recovered by the detector if $k_f = 100\pi$ rad/V, assuming that the message signal is a voltage signal (i.e., measured in volts).

2. **[boldface text added or modified 10/17/25:]** A direct digital synthesis (DDS) system has been designed with a clock frequency of 10 MHz. **The phase accumulator has a length of n = 24 bits, but the address register for the sine wave look-up table (ROM) is truncated to N = 16 bits.** The ROM stores only the first quarter cycle of sine wave values, which correspond to phase values from 0 to 0.5π radians, so two of the **16** address bits **in the ROM** are used to determine the phase quadrant. **The DAC is therefore 14 bits wide.**

- **a.** Find the required phase increment M (i.e., the ROM address step size, an integer) to produce an output signal with a frequency as close as possible to 1.5 MHz.
- **b.** Find the tuning frequency increment of the DDS system. That is, find the frequency change that occurs when the value of M is incremented or decremented by one.

3. A frequency division multiplexing (FDM) scheme has 100 adjacent audio channels, each of which is bandlimited to 3.0 kHz (2.4 kHz plus a 600-Hz "guard" band), for a total bandwidth of 300 kHz. Each channel contains an SSB-SC signal (USB), and the modulating audio signals have approximately equal amplitude. The multiplexed signal is applied to an FM modulator with a peak frequency deviation of 1.0 MHz. Determine the bandwidth of the composite FM signal.

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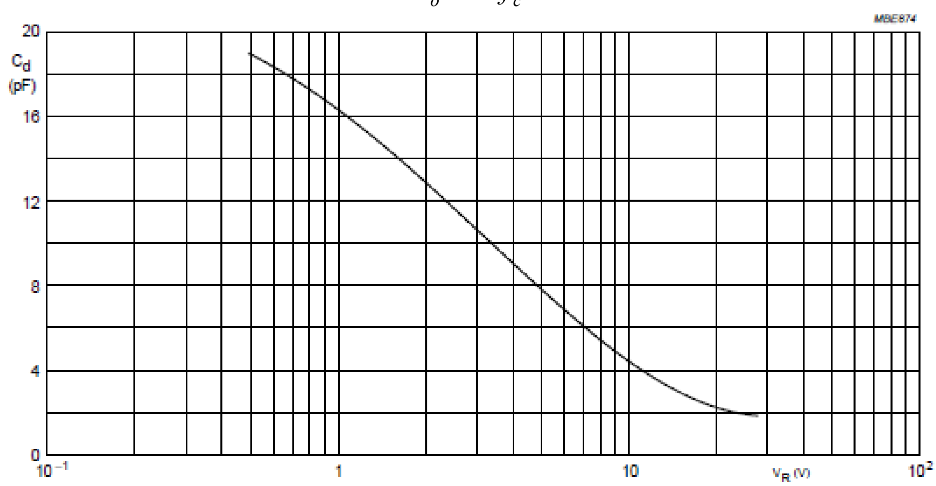
4. An FM modulator for WVBU is to be designed using the direct method with a varactor diode (voltage-controlled capacitance) in parallel with a 100-nH inductor L and a 20-pF fixed capacitor C_f. The desired carrier frequency is 90.5 MHz, and the desired peak frequency deviation is 75 kHz. An NXP Semiconductors type BB135 varactor diode will be used, which has the capacitance vs. reverse bias voltage relationship shown in the figure below. Find the average reverse bias voltage V_R and the peak modulating voltage magnitude $v_{r,pk}$ (labeled m_p in the textbook) that must be applied to the varactor diode to achieve the specifications. *Hint*: The value of the fixed capacitor C_f has been chosen so that the average value of the varactor diode capacitance is near the most linear part of the diagram below. You may assume that the approximation given below is valid, where ΔC = maximum change in the varactor diode's capacitance corresponding to the expected range of amplitudes for the message signal, C_o = average varactor diode capacitance, Δf = peak frequency deviation, and f_c = carrier frequency.

$$\frac{\Delta C}{C_o} \approx \frac{2\Delta f}{f_c}$$



***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. [adapted from Prob. 5.6-3 of Lathi & Ding, 4th ed.] Consider a shortwave radio that uses a superheterodyne architecture with a standard IF of 455 kHz. The receiver uses high-side injection, which means that the frequency range of the first local oscillator lies above the desired RF signal to be detected. The receiver is currently adjusted to pick up broadcast stations in the 31-meter band, which spans 9.4–9.9 MHz. The first stage of the receiver is a high-quality band-pass filter that greatly attenuates signals outside the 31-meter band.

- **a.** Determine the required frequency range of the local oscillator.
- **b.** Determine whether it is possible for the receiver to detect both a desired station and an image station that lie within the 31-meter band.

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2. The FM broadcast band spans the 88-108 MHz frequency range. The most common FM receiver architecture uses a first IF of 10.7 MHz and an LO range of 98.7–118.7 MHz. The image band therefore lies in the range 109.4–129.4 MHz, so it is necessary to use a tunable band-pass filter as the first stage of the receiver. Suppose that you have formed a new company that will market FM receivers with a higher first IF to relax the front-end filter requirements. Find the required IF and LO range that would be needed to move the image band to the range 300–320 MHz. Place the LO range between the FM broadcast band and the image band (i.e., use high-side injection).

3. An interesting approach for demodulating FM signals is to approximate a time derivative using a delay line as shown in the figure below. (An FM signal can be demodulated by passing it through a differentiator and then an envelope detector.) Recall that a first-order derivative in the time domain corresponds to multiplication by $j\omega$ in the frequency domain. The idea is that a derivative can be approximated by a finite difference, defined as

$$y(t) = \frac{dx(t)}{dt} \approx \frac{x(t) - x(t-\tau)}{\tau},$$

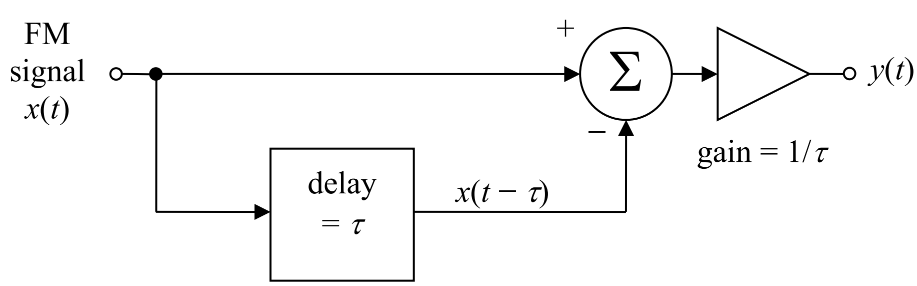
where τ is a short time delay. Taking the Fourier transform of both sides of the expression above yields

$$Y(\omega) = \frac{1}{\tau}\left[X(\omega) - e^{-j\omega\tau}X(\omega)\right] = \frac{X(\omega)}{\tau}\left(1 - e^{-j\omega\tau}\right).$$

If $\omega t << 1$, then the relationship can be approximated as

$$Y(\omega) \approx \frac{X(\omega)}{\tau}\left[1 - (1 - j\omega\tau)\right] = j\omega X(\omega).$$

Consider applying this approach to demodulate the WVBU signal at 90.5 MHz using ωt = 0.1 (so that the condition $\omega t << 1$ is satisfied). Find the required length of the delay line, and find the required gain in dB of the amplifier that follows the summing junction. Assume that the signal $x(t)$ is measured in volts, and assume that waves propagate at the speed of light in free space (3.0×10^8 m/s) along the delay line. Comment on whether the calculated delay line length and amplifier gain have practical values.



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4. A colleague has come up with an idea for generating an FM signal using a variation of the direct method. She wonders whether the variable pressure of sound waves could be used to modulate the spacing between the metallic plates of a capacitor. She proposes fixing one of the plates in place so that it is immovable and attaching the other plate to a diaphragm that will flex a small amount when exposed to sound waves. The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d},$$

where ε_r is the relative permittivity of the dielectric, ε_0 is the permittivity of free space (8.854×10^{-12} F/m), A is the area of one plate, and d is the separation distance between the plates.

To test the idea, you build an air-insulated capacitor with a nominal spacing of 0.5 mm between the plates. Each plate is a piece of metallic foil that is 1.4 cm square in size (i.e., 1.4 cm × 1.4 cm). One of the plates is affixed to a diaphragm.

- **a.** Find the nominal (unmodulated) value of the parallel-plate capacitor.
- **b.** Find the required value of the inductor that would be paired with the capacitor to resonate at a carrier frequency of 90.5 MHz.
- **c.** Find the maximum variation in the relative spacing $\Delta d_{\max}$ between the capacitor plates about the nominal value that would be required to generate an FM signal with a peak deviation of 75 kHz. (The spacing would range from $d_{\text{nom}} - \Delta d_{\max}$ to $d_{\text{nom}} + \Delta d_{\max}$.) Also express the maximum variation in spacing as a percentage of the nominal spacing.
- **d.** Comment on whether you think that this idea might be feasible.