ECEG 470/670 Communication and Information Systems Fall 2025

Homework Assignment #4 – due via Moodle at 11:59 pm on Monday, Oct. 27, 2025

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

The first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. [adapted from Prob. 6.2-4 of Lathi & Ding, 4th ed.] A message signal $m(t)$ with a time average power of 20 mW (i.e., $\overline{m^2(t)}$ = 20 mW) is applied to a quantizer with range limits of ±1.0 V. Uniform quantization is applied to the signal to encode it using PCM without compression. The SQNR (signal-to-quantization noise ratio) is required to be at least 43 dB. Find the minimum number of bits required to encode the signal, and determine the actual SQNR obtained with the final design. (It should be greater than 43 dB!)

2. A binary channel that can accommodate a bit rate of up to 52 kbps (kilobits per second) is available for use with a PCM-encoded voice signal that is bandlimited to 3.2 kHz. The signal is to be sampled at a rate that is 30% above the Nyquist limit. Find the maximum number of bits and the corresponding number of quantization levels that could be used to encode the signal using uncompressed PCM.

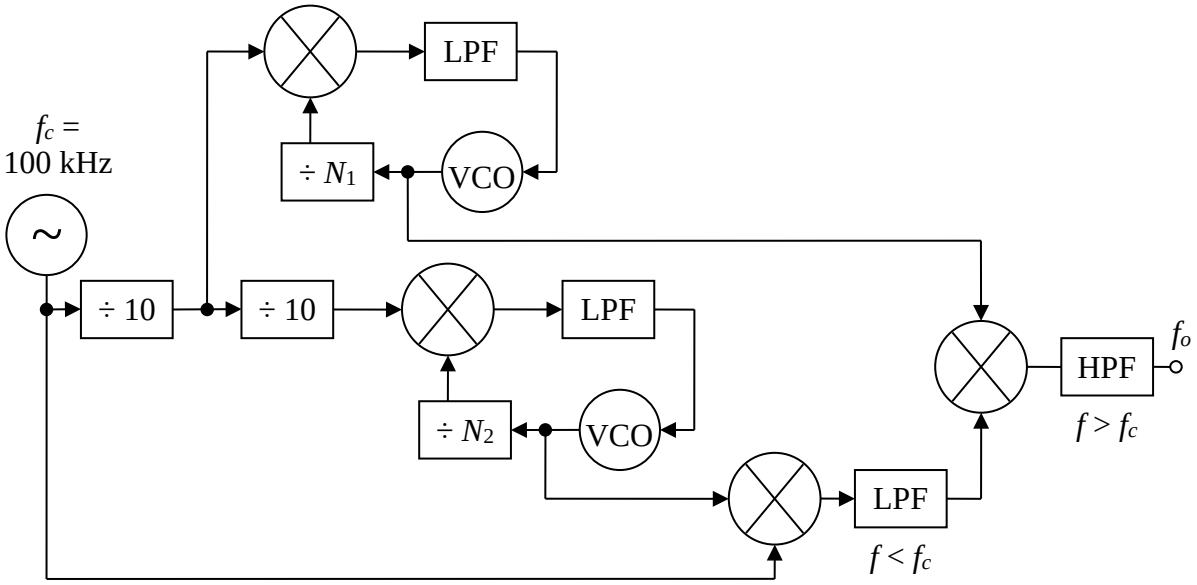
3. A compact disc (CD) recording system samples each channel of a stereo signal using a 16-bit analog-to-digital converter (ADC) operating at a sample rate of 44.1 kHz. Remember that a stereo recording system has two independent channels. No compression is used.

- **a.** Determine the output SQNR (signal-to-quantization noise ratio), expressed in dB, for a full-scale sinusoid, that is, one whose amplitude variation extends over the full input range of the ADC.
- **b.** The digital bit stream produced by the recording system is augmented by error correction and other control data so that the information overhead is 100%. That is, for every bit of digital music data, there is one bit of error correction and control data. Find the output bit rate of the recording system.
- **c.** The CD can record roughly an hour's worth of music. Find the number of bytes stored on a full CD (1 byte = 8 bits).

(*continued on next page*)

4. The diagram below depicts a possible architecture for a tunable frequency synthesizer (digitally controlled oscillator) based on a double-loop PLL. The 100-kHz oscillator is crystal controlled and therefore very stable. The blocks labeled with division amounts are frequency dividers, which are nonlinear circuits that produce at their output a signal with a frequency equal to the input frequency divided by the indicated amount. In the case of the dividers within the two PLL loops, N_1 and N_2 are user-selectable integer values between 1 and 10. The low-pass filter labeled $f < f_c$ and the high-pass filter labeled $f > f_c$ each have a cut-off frequency equal to the crystal oscillator frequency of f_c = 100 kHz.

a. Find the output frequency f_o when $N_1 = 3$ and $N_2 = 7$.
b. Determine the frequency range of the system and frequency tuning increment.



***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. An analog signal will be quantized and transmitted using a PCM system. The signal has positive and negative values and zero average value. Each sampled value of the signal must be accurate to within ±0.5% of the peak-to-peak full-scale range of the quantizer. The peak-to-peak range extends from $-m_p$ to $+m_p$, where m_p is the peak magnitude that can be handled by the quantizer. Find the minimum number of bits that each sample must contain to meet the specification.

2. An audio signal is passed through a band-pass filter with cut-off frequencies of 300 Hz and 3000 Hz. The signal is sampled at a rate of 8.0 kHz to generate a PCM signal. The application requires that the SQNR (signal-to-quantization noise ratio) be 30 dB or greater for a full-scale signal (i.e., one with a normalized power level equal to $0.5m_p^2$). Assuming uniform quantization, find the minimum number of quantization levels and the corresponding number of bits required to meet the specification.