ECEG 470/670 Communication and Information Systems Fall 2025

Homework Assignment #5 – due via Moodle at 11:59 pm on Tuesday, Nov. 11, 2025

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

The first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. [Adapted from Prob. 6.2-7 of Lathi & Ding, 4^{th} ed.] A message signal *m*(*t*) with a time average power of 20 mW is applied to a quantizer with range limits of ±1.0 V. A mu-law compandor with $\mu = 100$ encodes the signal using nonuniform PCM. The SQNR (signal-to-quantization noise ratio) is required to be at least 43 dB. Find the minimum number of bits required to encode the signal, and determine the actual SQNR obtained with the final design. (It should be greater than 43 dB!) This problem is almost the same as Graded Prob. 1 from HW #4 except that companding is used instead of uniform quantization.

2. An audio signal is passed through a band-pass filter with cut-off frequencies of 300 Hz and 3000 Hz and is then sampled at a rate of 8 kHz to generate a PCM signal. The SQNR (signal-to-quantization noise ratio) must be 30 dB or greater for a full-scale signal (i.e., one with a mean squared amplitude equal to $0.5m_p^2$). Mu-law companding will be used with $\mu = 255$. Find the SQNR and the minimum required system bandwidth.

3. [Adapted from Prob. 5.5-1 of Lathi & Ding, 6^{th} ed.] A simple DPCM encoder is applied to the message signal given below left. The sampling frequency is 2.5 kHz, and a first-order predictor is used that is described by the expression below center that results in the prediction error (difference signal) given below right.

$$m(t) = 0.050\cos(1{,}000\pi t)\ \text{V} \qquad \hat{m}_q[k] = m[k-1] \qquad d[k] = m[k] - \hat{m}_q[k] = m[k] - m[k-1]$$

a. Using the substitutions and the trigonometric identity given below, determine the peak value of *d*[*k*] (equal to d_p). Solve the problem symbolically and then substitute the numerical values $\omega = 1{,}000\pi$ and $T_s = 400$ µs. Note that $t = kT_s$ or $(k-1)T_s$.

$$a = \omega t - \frac{\omega T_s}{2} \qquad b = \frac{\omega T_s}{2} \qquad 2\sin a \sin b = \cos(a-b) - \cos(a+b)$$

b. Evaluate the improvement in the signal-to-noise ratio (in dB) than can be achieved by this DPCM encoder over a standard PCM encoder, assuming that m_p and d_p are equal to the peak amplitudes of the message signal and the difference signal, respectively.

(*continued on next page*)

4. [Adapted from Prob. 5.6-2 of Lathi & Ding, 6th ed.] The message signal described by the expression below is transmitted via a delta modulation (DM) system using a sampling frequency of 32 kHz.

$$m(t) = 8.0\cos(700\pi t) + 2.0\cos(2{,}400\pi t) + 0.44\cos(4{,}800\pi t) + 0.037\cos(6{,}800\pi t)$$

a. Determine the minimum step size E necessary to avoid DM slope overload.

b. Calculate the minimum average quantization noise power based on the answer obtained in Part **a**. The granular noise level is given by the expression below.

c. Find the signal-to-noise ratio of the DM system. Over long periods of time relative to the longest sinusoidal cycle, the time-average power of a sum of sinusoids is approximately equal to the sum of the time-average powers of the individual sinusoids. The signal and noise power levels are relative to the same impedance.

$$N_o = \frac{E^2 B}{3 f_s}$$

***Ungraded Problem*:**

The following problem will not be graded, but you should attempt to solve it on your own and then check the solution.

1. A 350-Hz sinusoid with an amplitude of 2.0 V, represented by the expression below, is sampled at a rate of 8.0 kHz starting at time $t = 0$. An excerpt of the signal is plotted below with samples 20 through 23 indicated by dots.

$$m(t) = 2.0\cos(700\pi t)\ \text{ V}$$

a. Predict the value of sample #23 using the predictor based on the first two terms of the Taylor series approximation of $m(t)$ and defined by the expression below, and calculate the error between the predicted value and the actual value.

$$m[k+1] = 2m[k] - m[k-1]$$

b. Predict the value of sample #23 using the predictor based on the first three terms of the Taylor series approximation of $m(t)$ and defined by the expression below, and calculate the error between the predicted value and the actual value.

$$m[k+1] = 2.5m[k] - 2m[k-1] + 0.5m[k-2]$$