ECEG 470/670 Communication and Information Systems Fall 2025

Homework Assignment #6 – due via Moodle at 11:59 pm on Tuesday, Dec. 9, 2025

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

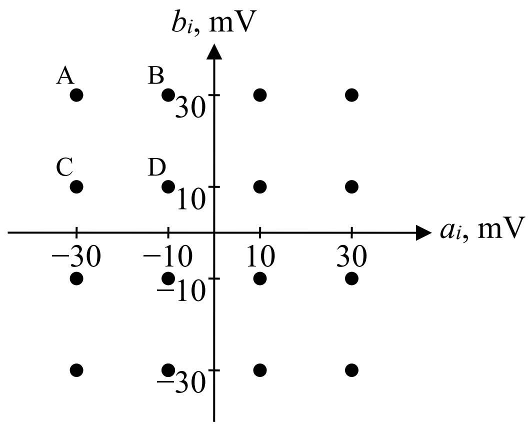
The first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

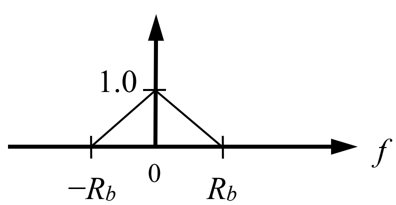
1. A mathematical expression for a 16-QAM signal is given by

$$\phi_{16-QAM}(\omega) = a_i p(t)\cos\omega_c t + b_i p(t)\sin\omega_c t, \qquad i = 1, 2, 3, 4,$$

where the constants $\{a_i\}$ and $\{b_i\}$ each have the four different possible values given in the constellation diagram shown below; $p(t)$ is the pulse shape in the time domain (with a peak value of 1); and ω_c is the RF carrier frequency. Find the amplitude and phase of the 16-QAM signal for each of the four states in the constellation diagram labeled A through D.



2. A baseband signaling pulse has the Fourier transform shown in the plot below, where R_b is the bit rate. The inverse Fourier transform (i.e., the pulse shape in the time domain) is given by the expression below, where $T_b = 1/R_b$. Show that the pulse satisfies Nyquist's first criterion for zero ISI.



$$p(t) = \frac{1}{T_b}\operatorname{sinc}^2\left(\frac{\pi t}{T_b}\right)$$

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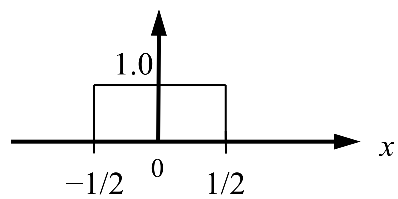
3. The raised cosine spectrum with roll-off factor $r = 1$, which satisfies Nyquist's first criterion for zero ISI, has the spectrum $P(f)$ given below and the corresponding time-domain pulse shape $p(t)$ shown next to it, where R_b is the bit rate and $T_b = 1/R_b$. The unit rectangular function $\prod(x)$ is plotted below the expressions for $P(f)$ and $p(t)$. Show that the envelope (maximum magnitude) of $p(t)$ rolls off as $1/t^3$ as $t \to \infty$.

$$P(f) = \cos^2\left(\frac{\pi f T_b}{2}\right)\prod\left(\frac{f}{2R_b}\right) \qquad p(t) = R_b \frac{\cos(\pi R_b t)}{1 - 4R_b^2 t^2}\operatorname{sinc}(\pi R_b t)$$

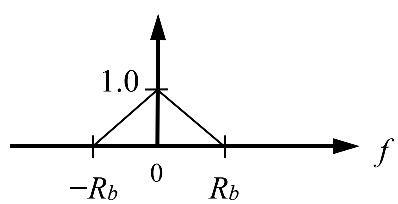


***Ungraded Problems*:**

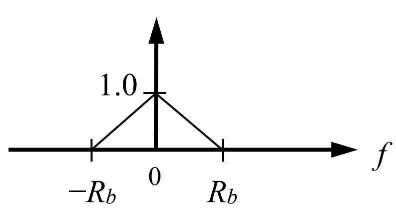
The following problems will not be graded, but you should attempt to solve it on your own and then check the solution.

1. Consider a baseband signaling pulse that has the Fourier transform shown in the plot below, where R_b is the bit rate. As explained in Section 6.3.2 of the textbook (Lathi & Ding, 6th ed.), one way to prove that a pulse satisfies Nyquist's first criterion for zero ISI is to show that the Fourier transform $P(f)$ of the pulse satisfies the summation given below. Show that the pulse considered here satisfied the criterion.

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = \text{constant}$$



2. A baseband signaling pulse has the Fourier transform shown in the plot below, where R_b is the bit rate. The inverse Fourier transform (i.e., the pulse shape in the time domain) is given by the expression below, where $T_b = 1/R_b$. The pulse shape will be used in a baseband data transmission system with polar signaling. Find the approximate channel bandwidth that would be required if the bit rate is to be 10 Mbits/s.



$$p(t) = \frac{1}{T_b}\operatorname{sinc}^2\left(\frac{\pi t}{T_b}\right)$$

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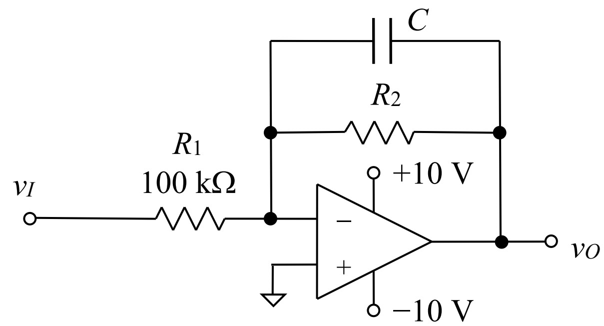
3. The circuit depicted below is a low-pass filter that is to be used as an amplifier-integrator in a delta modulation (DM) system. The message signal is audio that has been bandlimited to 300–3,000 Hz. The filter is to have a cut-off frequency of 30 Hz (well below 300 Hz) and a gain magnitude of approximately 1.0 at 300 Hz so that the circuit acts like an integrator (gain proportional to $1/j\omega$) over the message signal's spectrum. To establish a sufficiently high input resistance, R_1 is set to 100 kΩ. Find the values of R_2 and C required to meet the specifications. Voltages v_I and v_O are in phasor form. *Hint*: Well above the cut-off frequency, the gain of the circuit is given by the approximation below.

$$\frac{v_O}{v_I} \approx -\frac{1}{j\omega R_1 C}$$



Note: Some of the problems in this homework assignment have been adapted from a published source. The source is not cited here to preserve the integrity of the assignment. Source information is available upon request.