ECEG 497 — Wireless System Design — Spring 2023

Final Exam General Information

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-99.
2. You will be allowed to use up to **four** 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you later if you wish to have them back.
4. **You may not leave the exam room before completing your exam without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.** If you have a medical condition that might require you to leave the room, you must notify me before the exam begins. Only one student at a time may be absent from the room and must leave any electronic devices in the room.

The final exam will take place **8:00–11:00 am on Wednesday, May 10 in Academic East 225**.

Your graded final exam will not be returned to you, nor will the solutions be posted. However, you may make an appointment with me at any time to review your final exam and discuss your performance on it. I will keep your final exam at least until you graduate from Bucknell.

A list of topics to be covered on the exam begins on the next page.

Review Topics for Final Exam

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. For background purposes, you should be familiar with the topics on the review sheets for the previous exams in addition to those listed below.

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook and the supplemental readings are the final authorities in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate and authoritative information when preparing for your exam.

Nonlinear behavior of amplifiers and systems (intermodulation distortion, or IMD)

- representation of output signal as Taylor series (Maclaurin series, technically)
  $v_{out}(t) = a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + \cdots$, where a_1, a_2, a_3, … are constants
  Generally, $|a_1| > |a_2| > |a_3| > \cdots$
- 1st-order products are linear outputs
- 2nd-order, 3rd-order, etc. are intermodulation products
- dB levels of 2nd-order products increase *two* times as fast as those of 1st-order products as input power (in dBm) increases ($P_{o2} \sim P_{in}^2$)
- dB levels of 3rd-order products increase *three* times as fast as those of 1st-order products as input power (in dBm) increases ($P_{o3} \sim P_{in}^3$)
- third-order intercept point (TOI or IP3 or P_3), referred to input or output (assume output if not specified)
- 1 dB or 3 dB compression point (or compression level)
- IMD products usually troublesome only when they emerge from noise floor of amplifier or system
- 3rd-order products are of most concern because the signals that cause them can pass through front-end filter of receiver or amplifier

Minimum discernable (or detectable) signal (MDS)

- defined as the "signal power applied to the input [that] results in a specified SNR at the output, assuming perfect impedance matching at [the] input and output, and assuming the bandwidth of the two-port is at least as large as the bandwidth of the signal." (Ellingson, *Radio Systems Engineering*, 2016)
- min. acceptable SNR is often specified as 0 dB but could be any value, depending on modulation, signal integrity requirements, etc.
- multiple definitions of MDS, but the two most widely used are (assuming SNR = 0 dB)

  $$\text{MDS} = kT_{eq}B = kT_o\left(F-1\right)B \qquad \text{(assumes noise-free input)}$$

  $$\text{MDS} = kT_oB + kT_{eq}B = kT_oFB \qquad \text{(assumes standard noise input),}$$

  where k = Boltzmann's constant (1.38×10^{-23} J/K), T_o = standard temperature (290 K), and B = bandwidth of stage or system (narrowest)
- if reference SNR is not 0 dB, then expressions for MDS above must be multiplied by SNR expressed in factor (not dB) form; alternatively, SNR in dB is added to MDS in dBm

- related to noise figure by (using definition with standard input noise):

  $$\text{MDS}[\text{dBm}] = 10\log\left(\frac{kT_o}{0.001}\right) + 10\log(B) + \text{NF} + \text{SNR}[\text{dB}]$$

- $10\log\left(\frac{kT_o}{0.001}\right) = -174\text{ dBm}$

Dynamic range of amplifiers and receiver systems

- blocking dynamic range: $\text{BDR}[\text{dB}] = P_{in,1\,\text{dB}} - \text{MDS}[\text{dBm}]$, where $P_{in,1\text{ dB}}$ is the input power level at the 1 dB output compression point
- "two-tone" third-order, or spurious-free, dynamic range:

  $\text{SFDR} = \frac{2}{3}\left(\text{IP3}_{in} - \text{MDS}\right)$, where IP3_{in} is the third-order input intercept point;

  all quantities are in dBm or dB
- remember that MDS is defined relative to an appropriate SNR (usually, but not always, SNR = 0 dB)

Time-average Poynting vector [not specifically covered on exam; the information in this section is provided for background information]

- definition: $\mathbf{S}_{av} = \frac{1}{2}\text{Re}\left\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right\}$, if electric and magnetic fields are in peak units and expressed as phasors
- gives the power density per unit area of an EM wave (unit is the W/m^2)
- points in direction of power flow and propagation of phase fronts (in lossless media)

Common characteristics of radiated far fields of all antennas: [provided for background info]

- the $\frac{e^{-jkR}}{R}$ factor, which implies spreading spherical waves
- propagation in $\hat{\mathbf{R}}$ direction (if antenna is centered at origin)
- speed of propagation is $\frac{1}{\sqrt{\mu\varepsilon}}$ (speed in the surrounding medium; true for *all* TEM waves, which includes fields radiated by *all* antennas)
- electric and magnetic fields are proportional to input current (true for *all* antennas fed by a transmission line)
- $\tilde{\mathbf{E}} \perp \tilde{\mathbf{H}}$, $\tilde{\mathbf{E}} \perp \mathbf{S}_{av}$, and $\tilde{\mathbf{H}} \perp \mathbf{S}_{av}$ (where $\mathbf{S}_{av}$ = time-average Poynting vector; true for *all* TEM waves)
- electric and magnetic fields are in phase if η is purely real (true for *all* TEM waves)
- $\frac{|\tilde{\mathbf{E}}|}{|\tilde{\mathbf{H}}|} = \eta$ (true for *all* TEM waves)

Far fields of half-wave dipole w/peak input current I_m & oriented along z-axis: [provided for background information]

$$\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}}\frac{j\eta\tilde{I}_{in}}{2\pi}\frac{e^{-jkR}}{R}\frac{\cos\left(0.5\pi\cos\theta\right)}{\sin\theta}, \quad \tilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}}\frac{j\tilde{I}_{in}}{2\pi}\frac{e^{-jkR}}{R}\frac{\cos\left(0.5\pi\cos\theta\right)}{\sin\theta}$$

Radiation pattern [provided for background information]

- plot of $|\mathbf{S}_{av}|$ (sometimes normalized), directivity, or gain vs. θ and/or ϕ
- normalized power pattern:

  $$F(\theta,\phi) = \frac{|\mathbf{S}_{av}|}{S_{\max}},$$

  where S_{max} is the maximum Poynting vector magnitude in any direction (θ, ϕ) at a particular distance R
- usually plotted using a dB (or dBi, for directivity and gain) scale
- interpretation of radiation pattern plot (either in terms of actual gain/directivity or the normalized power pattern)
- determination of relative power in various directions

Directivity and gain [provided for background information]

- concept of isotropic radiator
  - hypothetical antenna that radiates with equal intensity in all directions
  - radiated fields have no specific polarization (not realistic)
  - Poynting vector of isotropic radiator: $\mathbf{S}_{iso} = \hat{\mathbf{R}} \dfrac{P_{in}}{4\pi R^2}$, where P_{in} is input power to isotropic antenna, which is assumed to be lossless
- directivity calculated from power pattern

  $$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi} = \frac{S_{\max}}{|\mathbf{S}_{iso}|} = 4\pi R^2 \frac{S_{\max}}{P_{rad}} = 4\pi R^2 \frac{S_{\max}}{P_{in}} \text{ (assumes no power losses)}$$

Gain (G) and efficiency (ξ) [provided for background information]

- $G = \xi D$, where D is the directivity
- $P_{rad} = \xi P_{in}$
- $\xi = \dfrac{R_{rad}}{R_{rad} + R_{loss}}$, if R_{rad} and R_{loss} are in series in the input impedance model
- loss resistance usually represents finite conductivity of antenna structure and/or ground beneath it, but other factors can contribute to loss as well
- calculation of power density of antenna radiation (does not include xmsn line losses):

  $$S_{\max} = \frac{P_{in}G}{4\pi R^2} = \frac{P_{in}\xi D}{4\pi R^2}$$
- gain and directivity are usually expressed in dBi (dB relative to an isotropic radiator):

  D[dBi] = 10 log(D) $D = 10^{D[\text{dBi}]/10} = 10^{0.1D[\text{dBi}]}$

  G[dBi] = 10 log(G) $G = 10^{G[\text{dBi}]/10} = 10^{0.1G[\text{dBi}]}$

  $G = \xi D$ → 10 log(G) = 10 log(ξ) + 10 log(D) → G[dBi] = ξ[dB] + D[dBi]

  where ξ, because it is less than 1, has a negative value in dB
- directivities of short dipole, Hertzian dipole, and small loop are all 1.5 (1.76 dBi) because normalized power patterns are all $\sin^2\theta$
- directivity of half-wave dipole is 1.64 (2.15 dBi)
- dBi unit (gain/directivity in dB referenced to isotropic radiator) vs. dB unit

Radiation resistance [provided for background information]

- input impedance of antenna: $Z_{in} = R_{rad} + R_{loss} + jX_{in}$,
  where R_{rad} = radiation resistance, R_{loss} = loss resistance, X_{in} = input reactance
- R_{rad} is real part of equivalent input impedance that represents radiated power
- accounts for power delivered by transmission line that is radiated by antenna

- half-wave (nominally) dipoles
  - current distribution is a half-sinusoid with peak at feed point
  - capacitive input reactance if shorter than first resonant length
  - inductive input reactance if longer than first resonant length
- half-wave dipole: $R_{rad} = 73\ \Omega$ (ideal half-wave dipole isolated in free space)
- quarter-wave monopole: $R_{rad} = 36.5\ \Omega = (73\ \Omega)/2$ (ideal monopole over perfectly conducting ground plane of infinite extent)

Specialized computational methods like the one used in *EZNEC* are required to find accurate current distributions along real antennas.

Phased array antennas

- multiple elements, each excited by a current or voltage of unique amplitude and phase; elements are usually identical
- complex current at input terminals (a.k.a. excitation coefficient): $I_n = |I_n| e^{j\phi_n}$
- concepts of element pattern, array factor, and pattern multiplication
- array factor is either the field pattern or the power pattern of array of isotropic elements; pay attention to context
- array factor for elements uniformly spaced along z-axis (power pattern form):

  $$AF(\theta) = \left| \sum_{n=0}^{N-1} |I_n| e^{j\phi_n} e^{jknd\cos\theta} \right|^2,$$

  where d = element spacing and $k = 2\pi/\lambda$.
- for arrays spaced along other coordinate axes, interpret θ as angle measured from axis
- amplitude distribution: variation of excitation magnitudes $|I_n|\big|_{n=0,1,2,\ldots}$ with location
- phase distribution: variation of excitation phases $\phi_n\big|_{n=0,1,2,\ldots}$ with location
- important special case: N-element array with uniform (equal-amplitude; i.e., $|I_n|$ = constant) excitation, uniform spacing d, and constant interelement phase shift $\Delta\phi$
  - individual excitation phases: $\phi_n = n\Delta\phi$, where n = 0, 1, 2, 3, …
  - *normalized* array factor becomes $AF_{norm}(\theta) = \dfrac{\sin^2\left[0.5N\left(kd\cos\theta + \Delta\phi\right)\right]}{N^2 \sin^2\left[0.5\left(kd\cos\theta + \Delta\phi\right)\right]}$

    Note: The textbook by Ulaby and Ravaioli, 7th ed., defines the interelement phase shift as δ, which is equal to $\Delta\phi$ here
  - to steer beam to $\theta = \theta_o$ direction, set $\Delta\phi$ using $\Delta\phi = -kd\cos\theta_o$
- broadside direction: perpendicular to array axis
- endfire direction: along array axis (applicable to Yagi-Uda arrays)
- grating lobes: appear at angles θ_G that satisfy $kd\cos\theta + \Delta\phi = \pm n2\pi$, where n = 1, 2, 3, … (but not n = 0; that case corresponds to the main beam) Physically meaningful angles must fall within the range $0 \le \theta \le 180°$ since θ is not defined outside that range; that is, any θ_G values that fall outside that range are not "visible."

Yagi-Uda arrays

- multiple straight elements in parallel; lengths are nominally half-wave
- one element is driven, others are parasitically coupled
- reflector is slightly longer than driven element and has inductive self-impedance
- directors are slightly shorter than driven element and have capacitive self-impedances
- radiation resistance of driven element is usually very different from 73 Ω (the value for an isolated half-wave dipole) due to mutual coupling with other elements; could require matching network

- mutual impedance matrix relates currents to voltages at "ports" of all elements. (The centers of the parasitic elements can be thought of as ports that are short-circuited so that their port voltages are zero but their port currents are nonzero.):

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 + \cdots + Z_{1N}I_N \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 + \cdots + Z_{2N}I_N \\ &\vdots \\ V_N &= Z_{N1}I_1 + Z_{N2}I_2 + \cdots + Z_{NN}I_N \end{aligned} \quad \rightarrow \quad \begin{aligned} 0 &= Z_{11}I_1 + Z_{12}I_2 + \cdots + Z_{1N}I_N \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 + \cdots + Z_{2N}I_N \\ 0 &= Z_{31}I_1 + Z_{32}I_2 + \cdots + Z_{3N}I_N \\ &\vdots \\ 0 &= Z_{N1}I_1 + Z_{N2}I_2 + \cdots + Z_{NN}I_N \end{aligned}$$

  where only the driven element (usually port 2) voltage is nonzero, so $V_1 = V_3 = V_4 = \ldots = V_N = 0$
- mutual admittance matrix is an alternate expression of the relationships between port currents and voltages:

$$\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 + \cdots + Y_{1N}V_N \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 + \cdots + Y_{2N}V_N \\ &\vdots \\ I_N &= Y_{N1}V_1 + Y_{N2}V_2 + \cdots + Y_{NN}V_N \end{aligned} \quad \text{reduces to} \quad \begin{aligned} I_1 &= Y_{12}V_2 \\ I_2 &= Y_{22}V_2 \\ &\vdots \\ I_N &= Y_{N2}V_2 \end{aligned}$$

  because $V_1 = V_3 = V_4 = \ldots = V_N = 0$ if only element 2 is driven (has nonzero voltage). The quantity Y_{i2} for $i = 1, 2, 3, \ldots, N$ is the mutual admittance between element 2 and the ith element with all of the parasitic elements in the array present. (For parasitic elements, $V_i = 0$ for $i \neq 2$.)
- mutual admittance matrix is the inverse of the mutual impedance matrix
- Computational methods like *EZNEC* are required to find highly accurate current distributions along the elements of real Yagi-Uda array antennas.
- Yagi-Uda arrays can also be made with nominally full-wave loop elements

<u>Relevant course material:</u>

HW:	#9
Mini-Projects:	#1
Reading:	Assignments from Apr. 7 through May 1, including the supplemental readings: Ulaby Sections 9-9 through 9-11 "Yagi-Uda Antennas"
Review sheets:	for Exams #1 and #2 (some past material might be referred to on final exam)

This exam will focus primarily on the course outcomes listed below and related topics.

6. Understand the relationship between minimum detectable signal (MDS), third-order intercept (TOI or IP3), and spurious-free dynamic range (SFDR) of an amplifier or receiver system.
8. Manually and/or numerically calculate important performance characteristics of commonly used antenna types. [focusing on phased arrays and Yagi-Uda arrays]

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.

Unfortunately, we did not have enough time to cover Outcome #7 (Use a Smith chart to plot impedances and to perform basic transmission line and matching network calculations.), so it will not be covered on the final exam.