Homework Assignment #3 – due via Moodle at 11:59 pm on Wednesday, Feb. 22, 2023

Instructions, notes, and hints:

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It is your responsibility to review the solutions when they are posted (including those for ungraded problems) and to understand and rectify any conceptual errors that you might have. You may contact me at any time for assistance.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems. One graded problem will be randomly selected for detailed evaluation; the others will be evaluated using a coarse rubric.

Graded Problems:

1. Find the capacitor and inductor values for the coupled-resonator filter shown below so that it has cut-off frequencies of 800 and 1000 MHz (to cover the cell phone band). Use an inductive reactance of 150 Ω at the center frequency. The system impedance is 50 Ω . Neglect all stray reactance effects.



2. Repeat the design task from the previous problem, but this time use the filter topology shown below. The inductive reactance should still be 150Ω at the center frequency.



3. Re-design the series-LC coupled-resonator filter in the previous problem so that the parallel element next to the load is an inductor instead of a capacitor.

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4. Shown below is a 20 dB T network attenuator designed for use in a 50 Ω system. Its purpose is to limit the power delivered to the load to a value 20 dB below the available power from the source while maintaining an input impedance of 50 Ω as seen by the source and an output impedance of 50 Ω as seen by the load. (The 50 Ω input and output impedances are obtained only if the source and load impedances are also 50 Ω .) Attenuators are widely used for RF measurements. Recall that the available power P_A is given by $|v_{th}|^2/4R_{th}$, where v_{th} is the Thévenin equivalent voltage in rms units and R_{th} is the Thévenin equivalent resistance. Find the available power of the signal source alone (represented by 1 V rms in series with 50 Ω in the circuit below), and then find the available power of the signal source and the attenuator circuit combined. Finally, compare the two values. That is, compare the power actually delivered to the load to the available power from the source alone and from the source plus attenuator. *Hint*: The specified attenuation of 20 dB is a very strong clue!



Ungraded Problems:

The following problem will not be graded. However, you should attempt to solve it on your own and then check the solution. Try not to give up too quickly if you struggle to solve it. Move on to a different problem and then come back to the difficult one after a few hours.

1. The formulas for attenuation derived in the lecture notes on filters assume that the source and load impedances are pure resistances. In general, however, both impedances can be complex as shown in the figure below left. Show that for the general case (for which $Z_L \neq Z_g^*$, where the asterisk indicates complex conjugation), the fraction of the available power P_A delivered to the load with no filter present is given by the expression shown below right. *Hint*: Find the power P_L delivered to the load in terms of the load current i_L . Remember that v_g and i_L are complex phasors. Available power is defined in one of the supplemental readings.



$$\frac{P_L}{P_A} = \frac{4R_g R_L}{\left|Z_g + Z_L\right|^2}$$

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2. Shown below is the same 20-dB T network attenuator analyzed in one of the graded problems. Using a nonideal resistor model consisting of 10 nH of inductance in series with each resistor in the attenuator and 1 pF of capacitance in parallel with each resistor in the attenuator (but not for the source and load resistances), find the actual attenuation of the network at operating frequencies of 1 MHz, 100 MHz, and 1 GHz. That is, compare the power actually delivered to the load to the available power from the source. For each case, express the attenuation in dB units. You might want to use numerical analysis software such as Matlab or Mathematica to solve this problem. *Hint*: The attenuation should be very close to 20 dB at 1 MHz.



3. Refer to Graded Problems 1 and 2 above. Use Matlab to plot the attenuation (in dB) vs. frequency over the 100–2000 MHz range for the filters you designed in the two problems. Calculate the network Q of the filter (i.e., the Q that determines the bandwidth), and use it to verify that $BW_f \approx f_0/Q$, where BW_f is the 3-dB filter bandwidth in Hz. For further practice, add the curve corresponding to the filter designed in Graded Problem 3.