ENGR 4/695 Advanced Topics in Engineering Mathematics Fall 2026

Homework Assignment #3 – due via Moodle at 11:59 pm on Tuesday, Sept. 29, 2026

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It is your responsibility to review the solutions when they are posted and to understand and rectify any conceptual errors that you might have. Please contact me if you need assistance.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. Diagonalize the matrix A given below if you can. If you can't, explain why. If possible, form the matrix X so that it is orthogonal. You must solve the problem by hand, but you may use *Matlab* or other software to verify your answer.

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$$

2. Find a 2×2 matrix A that has the eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 3$ and the corresponding eigenvectors given below. You must solve the problem by hand, but you may use *Matlab* or other software to verify your answer.

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

3. A matrix A and a possible LU factorization of it are given below. Use the LU factorization to solve the system $A\mathbf{x} = \mathbf{b}$ by hand for the vector $\mathbf{b}$ given below. You may use *Matlab* or other software to verify your answer.

$$A = \begin{bmatrix} 12 & 1 & 3 \\ 4 & -2 & 1 \\ -4 & 1 & 2 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ -1/3 & -4/7 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 12 & 1 & 3 \\ 0 & -7/3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} -15 \\ -12 \\ 3 \end{bmatrix}$$

(*continued on next page*)

4. For each of the matrices below, determine how many of the singular values would be zero if the matrix were decomposed using the SVD method. Explain how you arrived at your answer. You may use *Matlab* or other software to check your answers, but your solution to each part must indicate that you understand how to predict the number of singular values. You do *not* have to apply the SVD method to solve this problem; you should be able to predict the number of singular values in each case without using SVD.

$$\textbf{a.}\begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \qquad \textbf{b.}\begin{bmatrix} 4 & -1 \\ -1 & 0 \end{bmatrix} \qquad \textbf{c.}\begin{bmatrix} 2 & 1 & 1 \\ -4 & -2 & -2 \\ 8 & 4 & 4 \end{bmatrix} \qquad \textbf{d.}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

5. Starting with the result of the product AA^T, manually find expressions in terms of ε for the singular values and the orthogonal matrix U for the matrix B given below. There are four valid answers for the matrix U. Also find an expression in terms of ε for the condition number of the matrix. The quantity ε is a small value ($\varepsilon << 1$). Find the singular values and a valid form of U for the case when $\varepsilon = 0$, and briefly explain the implications of the results, especially how U changes (or not) for the cases when $\varepsilon = 0$ and $\varepsilon \neq 0$. You may check your answers using *Matlab*.

$$B = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}$$

***Ungraded Problems*:**

The following problems will not be graded. However, you should attempt to solve them on your own and then check the solutions. Try not to give up too quickly if you struggle to solve them.

1. Show that an orthonormal matrix that multiplies a vector does not change the length of the vector. That is, show that $\mathbf{x}$ and $Q\mathbf{x}$ (where Q is an orthonormal matrix) have the same length. Note that the length of a vector is given by $x = (\mathbf{x}^T\mathbf{x})^{1/2}$.

2. Show that an orthonormal matrix does not change the angle between two vectors (i.e., it does not change their relative orientation). To do this, show that $\theta_{xy} = \theta_{ab}$, where $\mathbf{a} = Q\mathbf{x}$ and $\mathbf{b} = Q\mathbf{y}$. Note that the angle between two vectors $\mathbf{x}$ and $\mathbf{y}$ is given by the expression

$$\cos\theta_{xy} = \frac{\mathbf{x}^T\mathbf{y}}{xy},$$

where x and y are the lengths of the vectors $\mathbf{x}$ and $\mathbf{y}$, respectively.

3. Starting with the result of the product A^TA, manually find the singular values of the following matrix. You may check your answer using *Matlab*.

$$\begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$