

Lecture Outline for Wednesday, Aug. 26, 2026

1. Linear systems, linear algebra, and matrix equations [review]
 - a. Multiple linear equations
 - b. Formulation: knowns and unknowns
 - c. Basic question: solutions...when and how?
2. Notation and conventions [review]
 - a. Scalars, vectors, and matrices
 - b. Size and shape (rows, columns)
 - c. Typography and notational items
3. Rules of linear algebra (matrix algebra) [review]
 - a. Addition, subtraction: size & shape matter; order does not
 - b. Multiplication
 - i. Many options (s-v, s-m, v-v, v-m, m-m; s = scalar, v = vector, m = matrix)
 - ii. Size & shape matter
 - iii. Possible loss of commutativity: direction and order matter
 - c. Division
 - i. Not always possible or even defined
 - ii. When defined, it is through multiplication by an inverse
4. Example #1: Expressing system of equations in matrix form

$$\begin{array}{rcl} 9x - 2y = 15 \\ 3x + 4y = 33 \end{array} \quad \rightarrow \quad \begin{bmatrix} 9 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 33 \end{bmatrix}$$

5. Example #2: Scaling the system of equations above by a factor of 3 and scalar multiplication of matrices and vectors

$$\begin{bmatrix} 27 & -6 \\ 9 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 45 \\ 99 \end{bmatrix} \quad \rightarrow \quad 3 \begin{bmatrix} 9 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} 15 \\ 33 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 9 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 33 \end{bmatrix}$$

6. Basic computations in linear algebra: $A\mathbf{x} = \mathbf{b}$ [review & preview]
 - a. A , $\mathbf{x}$ given: geometric transformations (images, outputs from inputs)
 - b. A , $\mathbf{b}$ given: system solution (inputs from outputs)
 - c. Size & shape matter in defining the solution
 - d. Geometrical interpretations
 - e. Unit vectors, projections, components, dot products

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Example Problems in Matrix Arithmetic

Use the following vector and matrix definitions:

$$\mathbf{c} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Compute the following quantities. You are encouraged to use *Matlab* if you have access to it.

1. $\mathbf{c}^T \mathbf{c}$, where superscript T indicates the matrix transpose operation. What attribute of $\mathbf{c}$ does this operation yield?
2. $\mathbf{b}^T \mathbf{c}$ What is the geometrical interpretation of this operation?
3. $\mathbf{b} \mathbf{c}^T$
4. $A \mathbf{b}$
5. AB
6. $I \mathbf{c}$ What is I usually called? What is the key property of I ?
7. Without resorting to computation, what are the results of the operations AI and IA ?
8. $I + \mathbf{b} \mathbf{c}^T$
9. A^T
10. What would be required of a matrix D so that D^2 (which equals DD), $D^T D$, and DD^T all yield the same result? Does the matrix A meet this requirement? Why or why not?