

Lecture Outline for Wednesday, Sept. 30, 2026

1. Applications of SVD

- a. Equivalent expressions for
- $\mathbf{x}$
- :

$$\mathbf{x} = V\Sigma^{-1}U^T\mathbf{b} \quad \rightarrow \quad \mathbf{x} = \sum_{j=1}^N \left(\frac{\mathbf{u}_j^T \mathbf{b}}{\sigma_j} \right) \mathbf{v}_j$$

In ill-conditioned systems, some $\mathbf{u}$ vectors are nearly orthogonal to $\mathbf{b}$. (In singular systems, some *are* orthogonal.) The corresponding singular values are very small, so those terms “corrupt” the solution $\mathbf{x}$. Workaround: Omit the problematic terms with small σ_j .

- b. Example #1: Compare eigenvalues to singular values of the symmetric matrix
- A
- (use
- Matlab*
- `eig`
- and
- `svd`
- commands):

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$

- i. Notice order of singular values
 - ii. Check orthogonality of columns of U and V
 - iii. Compare condition number (using *Matlab* command `cond`) to σ_1/σ_3
 - iv. What is the rank of this matrix?
 - v. What are the ranks of $\mathbf{u}_1\mathbf{v}_1^T$, $\mathbf{u}_2\mathbf{v}_2^T$, and $\mathbf{u}_3\mathbf{v}_3^T$?
- c. Example #2: Compare eigenvalues to singular values of the singular matrix A :

$$A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ 4 & 1 & -6 \end{bmatrix}$$

- i. Notice order of singular values
 - ii. Check orthogonality of columns of U and V
 - iii. Compare condition number (using *Matlab* command `cond`) to σ_1/σ_3
 - iv. What is the rank of this matrix?
 - v. What are the ranks of $\mathbf{u}_1\mathbf{v}_1^T$, $\mathbf{u}_2\mathbf{v}_2^T$, and $\mathbf{u}_3\mathbf{v}_3^T$?
- d. Example #3: Image processing/storage demonstration.