

**Homework Assignment #5 – due via Moodle at 11:59 pm on Wednesday, Apr. 8, 2026
[Graded Probs. 6 and 7 and Ungraded Prob. 1 added on 4/4/26; minor revisions 4/7/26]**

Instructions, notes, and hints:

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work. You may make reasonable assumptions and approximations to compensate for missing information, if any.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

Graded Problems:

1. The notes in the western chromatic scale (the one that includes all sharps and flats, 12 notes per octave) are:

C C[#] D D[#] E F F[#] G G[#] A A[#] B C

The “#” symbol indicates a sharp, which raises the note by a half step. In the equally tempered scale, a note raised by a sharp is enharmonic with (has the same pitch as) the next higher note lowered by a flat, indicated by the “b” symbol. Thus, D[#] is the same as E^b. By international agreement, the standard frequency of the note A₄ (the A above middle C and near the middle of a piano keyboard) is 440 Hz. Use arithmetical operations to find the frequencies associated with the following notes in the equally tempered scale. You may check your answers using Table 9.2 of the textbook or the “Frequencies for Equal-Tempered Scale” link at the course web site:

- a. D₁
 - b. E₃^b
 - c. G₅[#]
 - d. B₇
2. Find the frequency ratio of the interval C₄ : B₄ in the scale of equal temperament, where the first note (C₄) is lower in pitch than the second note (B₄). Remember that octave numbering begins with the C note. In the just intonation scale, the frequency ratio is 15/8. Your answer should be slightly different.
 3. Table 9.1 in the textbook (Rossing, et al., 3rd ed.) lists several commonly used musical intervals expressed in frequency ratio form and in cents. Show how the ratio expressed in cents is derived for the major third and the minor sixth in the just intonation scale; that is, calculate the numerical value yourself.

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4. In the early 1900s, there was a movement to define the pitches in the Western scales by basing the frequencies of the C notes on the powers of 2. For example, the frequency of C_3 would be 128 Hz (2^7 Hz), and C_4 would be 256 Hz (2^8 Hz). If this system had been adopted, then C notes would have been flat (lower in frequency) relative to the current system that is based on A_4 having a frequency of 440 Hz. Determine the difference in cents between C_3 and C_4 in the power-of-2 scale and the corresponding C notes in the current standard system based on the scale of equal temperament with $A_4 = 440$ Hz.
5. Find the tension (in newtons) required to tune the A_4 string on a violin to a fundamental frequency of 440 Hz. On a full-size violin, the length of the part of a string that vibrates (i.e., between the tuning nut and the bridge) is about 32.5 cm (0.325 m). The mass per unit length of the A_4 string is about 0.0025 kg/m.
6. **[Added 4/4/26]** Refer to Fig. 10.8 in the textbook (Rossing, et al., 3rd ed.). A performer pushes down on the E_5 string while bowing. A slow-motion video of the string reveals that the bend in the string caused by the bow completes a full round trip between the bridge and performer's finger in 1.01 ms (milliseconds, or thousandths of a second). Determine the fraction (or percentage – your choice) of the distance between the bridge and the nut where the performer pushed on the string. The full distance between the bridge and the nut is 32.5 cm (0.325 m).
7. **[Added 4/4/26]** Verify that each of the four strings on a standard violin (tuned to G_3 , D_4 , A_4 , and E_5) is tuned a fifth apart from its adjacent string by examining their frequency ratios. Assume that the strings are tuned to the standard frequencies in the scale of equal temperament.

Ungraded Problems:

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then return to the difficult one after a few hours.

1. **[Added 4/4/26]** Suppose that the A_4 string on a violin is plucked exactly $1/4$ of the distance between the bridge and the nut. The performer does not press on the string anywhere; the full length of the string vibrates. Determine the order (the corresponding multiplying integer) and frequency of the harmonic overtone that would *not* appear in the spectrum of the sound produced by the string.

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2. The expression below mathematically represents the case of a forward (subscript *fwd*) traveling wave and a reflected traveling wave (subscript *ref*) with the same amplitude A propagating along a string tied between two points. The waves on a string are transverse waves, so the displacement (motion) y of the string is perpendicular to the direction of propagation. The string is aligned along the x -axis, and the variable t is time.

$$y(x, t) = y_{fwd}(x, t) + y_{ref}(x, t) = A \cos(kx - \omega t) + A \cos(kx + \omega t),$$

where k is the wavenumber ($k = 2\pi/\lambda$) and ω is the radian frequency ($\omega = 2\pi f$). The variables λ and f are the wavelength in meters and frequency in hertz, respectively. Using the identity

$$\cos(a - b) + \cos(a + b) = 2 \cos(a) \cos(b)$$

with the substitutions $a = kx$ and $b = \omega t$, the expression for the total displacement y can be rewritten as

$$y(x, t) = 2A \cos(kx) \cos(\omega t) = 2A \cos(\omega t) \cos(kx),$$

which is a standing wave. As shown above, a standing wave can be thought of as two counter-propagating traveling waves added together. That concept helps to explain why standing waves (which represent vibrations) appear on strings and inside tubes.

Suppose that a certain string is 50 cm long and is vibrating at the first resonant frequency of 500 Hz. The maximum amplitude of the vibration (the greatest possible displacement) is 0.20 mm. Sketch the displacement of the full string (i.e., plot y vs. x) at the instances in time given by $t = 0$ ms, 0.25 ms, 0.50 ms, 0.75 ms, and 1.00 ms.