

**Homework Assignment #7 – due via Moodle at 11:59 pm on Monday, May 4, 2026*****Instructions, notes, and hints:***

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work. You may make reasonable assumptions and approximations to compensate for missing information, if any.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems:***

1. As explained in Sec. 14.3 of the textbook (Rossing, Moore, and Wheeler, 3<sup>rd</sup> ed.), the strings on a piano that are assigned to a single note (the unison strings for the note) are purposely mistuned by a few cents. Suppose that the strings for  $G_4$ , which has a frequency of 392.00 Hz in the scale of equal temperament, are mistuned by  $2\text{¢}$ . That is, the first string is tuned to exactly 392.00 Hz, the second  $2\text{¢}$  higher than the first, and the third  $4\text{¢}$  higher than the first. Find the frequency of the beats that a listener would hear when the piano key for  $G_4$  is pressed.
2. Imagine a piano in which all of the strings were made of the same material and had the same diameter and tension. The mass per unit length of all of the strings would be the same. If the longest strings, which produce the lowest note ( $A_0$ ), were 2.0 m long in this imaginary piano, find the length of the strings required to produce the highest note ( $A_7$ ). Would you consider this type of piano to be practical? Why or why not?
3. As shown in Fig. 12.10 of the textbook (Rossing, Moore, and Wheeler, 3<sup>rd</sup> ed.), the note  $C_4$  in the chalumeau register of a clarinet is fingered in the same way as the notes  $G_5$  in the clarion register and  $E_6$  in the altissimo register. Find the frequency ratios of  $G_5$  and  $E_6$  relative to  $C_4$  to the nearest integer, and briefly explain why they have those ratios.
4. Find the musical interval (e.g., major/minor third, fifth, sixth, etc.) between notes with the same fingering in the clarion and altissimo registers of the clarinet.

*(continued on next page)*

### **Ungraded Problems:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then return to the difficult one after a few hours.

1. The  $A_0$  string in a piano typically has a mass per unit length of 0.100 to 0.150 kg/m. Its length in a concert grand piano is about 240 cm. Find the required tension (in newtons) in the string to bring it into tune if the mass per unit length is 0.150 kg/m.
2. Repeat the previous problem for the  $A_4$  strings, which are about 34 cm long in a concert grand piano. The mass per unit length of the  $A_4$  strings is about 0.0056 kg/m.
3. The fundamental frequency of a steel  $A_4$  string in a piano is 440 Hz if it is not purposely mistuned. The second harmonic of the  $A_4$  string should therefore be 880 Hz. However, because of the steel string's stiffness, the second harmonic is slightly lower in frequency. Find how far off in cents the second harmonic is if the  $A_4$  string has the tension found in the previous problem. Assume that the mass per unit length and the total length of the string have the same values as in the previous problem. The  $A_4$  string typically has a diameter of around 1.0 mm. Steel has a Young's modulus value of around 200 GPa ( $200 \times 10^9$  pascals).
4. Examine the spectrum of partials produced by a piano when the  $C_1$  key is played shown in Fig. 14.7 of the textbook (Rossing, Moore, and Wheeler, 3<sup>rd</sup> ed.). The 9<sup>th</sup> and 17<sup>th</sup> harmonics are especially weak. Based on the spectrum, estimate the fraction (or percentage) of the string length at which the hammer strikes. *Hint*: Review Sec. 10.2 in the textbook.
5. Explain why the first two registers of a flute are an octave apart instead of a twelfth apart.