

**Policies and Review Topics for Final Exam**

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-99.
2. You will be allowed to use up to four 8.5 × 11-inch two-sided handwritten help sheets in addition to the sheets of graphs and formulas that I will provide to you. No photocopied material or copied and pasted text or images are allowed on the self-prepared help sheets. If there is a table, formula, or image from the textbook or some other source that you feel would be helpful and that is not included on the sheets that I will provide to you, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you either immediately or soon after the exam.
4. Use of a help sheet that is not completely handwritten will result in an automatic 5-point score reduction. Help sheets that are handwritten on a tablet and then printed are acceptable.
5. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
6. **You may not leave the exam room without prior permission except for an emergency or for an urgent medical condition. Please use the restroom before the exam.** If you are allowed to leave the room, you must leave your cell phone behind. Only one student at a time may be absent from the room.

The final exam is scheduled for **3:30–6:30 pm on Friday, May 8, 2026 in Breakiron 264**. You will have the full three hours to complete the exam.

Your graded final exam will not be returned to you, nor will the solutions be posted. However, you may make an appointment with me at any time to review your final exam and to discuss your performance on it. I will keep your final exam at least until you graduate from Bucknell.

A list of topics to be covered on the exam begins on the next page.

## Review Topics for Final Exam

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. **You should be familiar with the topics on the review sheets for the previous exams as well.**

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for exams.

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### Timbre

- Overall tone quality or tone “color” of a sound; it is the collective set of characteristics that help identify the type of sound and, often, its source
- Timbre depends mostly on frequency content (sound spectrum), but it can be affected by loudness and location within the spectrum
- Fourier analysis is the mathematical method of examining the spectrum of a sound.
- If the sound is periodic (i.e., the waveform repeats after a finite amount of time called the period  $T$ ), then it can be represented as a Fourier series. If  $p(t)$  is a periodic sound pressure wave as a function of time  $t$ , then the Fourier series representation is

$$p(t) = \sum_{n=1}^{\infty} [A_n \cos(2\pi n f_1 t) + B_n \sin(2\pi n f_1 t)],$$

where  $A_n$  and  $B_n$  are constants and  $f_1$  is the fundamental frequency, which determines the pitch of the sound and is equal to  $1/T$ .

- For this course, it is enough to know that when a tonal (not percussion) musical instrument plays a note, it produces a sound with a fundamental frequency and several harmonics. The relative intensities of the various harmonics play an important role in determining timbre. Only the first dozen or so harmonics in the Fourier series are significant (the higher-order harmonics are either too weak or too high in pitch to hear).

### The piano (Secs. 14.1–14.6)

- resonant frequency of a string (see Sec. 4.3 of textbook)

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}, \quad n = 1, 2, 3, \dots$$

where  $n$  = index of harmonic frequency (fundamental corresponds to  $n = 1$ ),  $L$  = effective length of string (in meters, m);  $T$  = tension of string (in newtons, N);  $\mu$  = mass per unit length (in kilograms per meter, kg/m)

- the square root of  $T/\mu$  in the equation above is the wave speed along the string
- basic structure of piano (keys, hammers, dampers, pin blocks, bridges, soundboard)
- a sturdy cast iron frame is needed to withstand the combined tension of 200+ strings with tensions that can exceed 1000 N (220 lb) per string for a total force of over 20 tons
- wide playing range of full-size piano: 88 keys; more than 7 octaves ( $A_0$  through  $C_8$ )
- piano strings are usually steel, although some can be brass, and the strings for the lowest notes are wire-wrapped steel

- inharmonicity of strings due to stiffness

$$f_n = nf_1 \left[ 1 - (n-1)^2 A \right] \quad A = \frac{\pi^3 r^4 E}{8TL^2},$$

where  $n$  = index of harmonic frequency (fundamental corresponds to  $n = 1$ ),  $r$  = radius of string (in meters, m);  $E$  = Young's modulus (about  $200 \times 10^9$  pascals for steel);  $T$  = tension of string (in newtons, N);  $L$  = length of string (in meters, m)

- inharmonicity can be reduced by using thin wires with high tension; wire wrapping of thick strings to reduce stiffness while maintaining mass per unit length
- *stretch tuning* is used to compensate somewhat for inharmonicity, although most people think that some inharmonicity should be present to create a "warm" sound
- tuning of unisons: most notes in a piano are produced by groups of two or three strings vibrating together, called *unisons*; unisons are purposely mistuned by 1–2 cents (hundredths of a semitone) relative to each other to extend the decay time of the vibrating strings
- if the unisons are too much out of tune, the "barroom piano" sound is the result
- the soundboard has many modes of vibration, each with a corresponding resonant frequency; the resonances are highly dependent on the soundboard's shape, structure, and stiffness and the number and locations of support ribs
- the harmonic content (spectrum) of each note produced by a piano depends on many factors; two key observations:
  - o the lower notes are richer in harmonics than the higher notes
  - o the harmonics associated with string vibration nodes near the hammer have greatly reduced relative amplitudes

#### Woodwind instruments (Chap. 12)

- woodwind family with bore types and reed types indicated
  - o clarinet (cylindrical bore, single reed)
  - o flute (cylindrical bore, air reed)
  - o tin whistle (cylindrical bore, air reed)
  - o saxophone (conical bore, single reed)
  - o oboe (conical bore, double reed)
  - o bassoon (conical bore, double reed)
  - o recorder (conical bore, air reed)
- resonant frequency of a closed cylindrical pipe (see Sec. 4.5 of textbook)

$$f_n = n \frac{v}{4L}, \quad n = 1, 3, 5, \dots$$

where  $n$  = index of harmonic frequency (fundamental corresponds to  $n = 1$ ) and the indices are *odd* integers,  $L$  = effective length of pipe (in meters),  $v$  = speed of sound in air at the ambient temperature (in meters per second)

- effective length of a pipe with a hole drilled into the side depends on the size of the hole relative to the pipe's diameter; the larger the hole, the closer the effective length of the pipe is to the distance to the hole (see Fig. 12.4 of textbook)

- tone-hole lattice
  - the open holes in a pipe beyond the section of the pipe in which the pressure standing wave is present (i.e., beyond the effective length with closed holes)
  - creates a “low-pass filter” effect; that is, harmonics above the cut-off frequency of the lattice are suppressed
  - cut-off frequency ( $f_c$ ) determination
 
$$f_c = 0.11 \frac{b}{a} \frac{c}{\sqrt{s(t+1.5b)}},$$

where  $a$  = radius of main pipe,  $b$  = radius of tone hole;  $c$  = speed of sound in air (about 344 m/s at room temp.),  $s$  = 1/2 of center-to-center distance between tone holes,  $t$  = thickness of pipe wall (depth of tone hole); all dimensions are in meters
- bore types
  - cylindrical: produces the fundamental resonant frequency of the pipe and the odd harmonics
  - conical: produces the fundamental resonant frequency of the pipe and the even *and* odd (i.e., all) harmonics
- sound generation in a clarinet with a single reed (see Fig. 12.2 in textbook)
  - initial puff of air through open reed produces a high-pressure region that propagates down the pipe
  - when the high-pressure region reaches the open end of the pipe, a rarefied (low-pressure) region “reflects” back up the pipe
  - the rarefied region reaches the reed, which helps to draw it closed
  - the rarefied region “reflects” from the reed and propagates back down the pipe toward the open end
  - when the rarefied region reaches the open end of the pipe, a high-pressure region “reflects” back up the pipe
  - the high-pressure region reaches the reed, which helps to push it open, and the cycle repeats
- registers and register holes
  - a register is a range of pitches associated with one of the modes of vibration
  - clarinet registers: chalumeau ( $n = 1$  mode), clarion ( $n = 3$ ), altissimo ( $n = 5$ ); the chalumeau and clarion registers are separated by a musical twelfth (3:1 frequency ratio) and the clarion and altissimo registers by a major sixth (5:3 ratio); separation between registers is a direct result of the cylindrical bore
  - although the flute has a cylindrical bore, its first two registers are separated by an octave (2:1 frequency ratio) because the flute pipe is open at both ends and thus resonates at *all* (not just odd) multiples of the fundamental frequency
  - first two registers of the conical bore instruments (saxophone, oboe, bassoon, recorder) are separated by an octave
  - register holes are located to “spoil” a resonance by leaking air (and thereby creating low pressure) where the standing wave for the mode should have high pressure; a register hole is also located near the node (zero-pressure point) of the harmonic associated with the desired register (e.g., the clarion register hole for a clarinet is located about 1/3 of the way from the mouthpiece)

Vibrations of membranes

- primary reason that circular drums do not have an identifiable pitch is that their vibration modes are not harmonically related; the overtones are not integer multiples of the fundamental frequency
- vibration nodes (regions that do not move for a particular mode) are lines in a circular membrane; the node lines can be radial or circular (see Fig. 13.9)
- timpani (kettledrums) sound more tonal than regular circular drums because the kettle is roughly hemispherical; the kettle shape shifts some of the overtones so that they are harmonically related to the fundamental
- the spectrum (combination of overtones) obtained from striking a circular membrane is highly dependent on where the membrane is struck

Relevant course material:

HW: #7  
Readings: Assignments from Apr. 15 through May 4  
Web Links: Parts of a Piano  
Piano Sound-Producing Mechanism  
Woodwind Instruments  
Flute Family  
Fingering Diagram for the Clarinet