

Karnaugh maps

3-variable case :

| A | B | C | Min term                      | F |
|---|---|---|-------------------------------|---|
| 0 | 0 | 0 | $m_0 = \bar{A}\bar{B}\bar{C}$ | 0 |
| 0 | 0 | 1 | $m_1 = \bar{A}\bar{B}C$       | 0 |
| 0 | 1 | 0 | $m_2 = \bar{A}B\bar{C}$       | 1 |
| 0 | 1 | 1 | $m_3 = \bar{A}BC$             | 1 |
| 1 | 0 | 0 | $m_4 = A\bar{B}\bar{C}$       | 1 |
| 1 | 0 | 1 | $m_5 = A\bar{B}C$             | 1 |
| 1 | 1 | 0 | $m_6 = AB\bar{C}$             | 0 |
| 1 | 1 | 1 | $m_7 = ABC$                   | 0 |

K-map is another way to organize:

| A \ BC | 00    |       | 01    | 11    | 10 |
|--------|-------|-------|-------|-------|----|
|        | $m_0$ | $m_1$ | $m_3$ | $m_2$ |    |
| 0      | $m_0$ | $m_1$ | $m_3$ | $m_2$ |    |
| 1      | $m_4$ | $m_5$ | $m_7$ | $m_6$ |    |

| A \ BC | 00                      |                   | 01                | 11          | 10 |
|--------|-------------------------|-------------------|-------------------|-------------|----|
|        | $\bar{A}\bar{B}\bar{C}$ | $\bar{A}\bar{B}C$ | $\bar{A}B\bar{C}$ | $\bar{A}BC$ |    |
| 0      | $\bar{A}\bar{B}\bar{C}$ | $\bar{A}\bar{B}C$ | $\bar{A}B\bar{C}$ | $\bar{A}BC$ |    |
| 1      | $A\bar{B}\bar{C}$       | $A\bar{B}C$       | $AB\bar{C}$       | $ABC$       |    |

Adjacent squares differ by only one variable, which is complemented in one square and uncomplemented in the other square.

$$\begin{aligned} \text{Ex: } m_5 + m_7 &= A\bar{B}C + ABC \\ &= AC(\bar{B} + B) = AC \cdot 1 = AC \end{aligned}$$

→ Single term with the variable that differs eliminated.

K-map for Function  $F$  in truth table:

| A \ BC |    |    |    |    |
|--------|----|----|----|----|
|        | 00 | 01 | 11 | 10 |
| 0      |    |    | 1  | 1  |
| 1      | 1  | 1  |    |    |

$F =$

Another Example:

| A \ BC |    |    |    |    |
|--------|----|----|----|----|
|        | 00 | 01 | 11 | 10 |
| 0      | 1  | 1  | 1  | 1  |
| 1      | 1  | 1  |    |    |

$F =$

| A \ BC |    |    |    |    |
|--------|----|----|----|----|
|        | 00 | 01 | 11 | 10 |
| 0      |    |    |    |    |
| 1      | 1  | 1  |    | 1  |

$F =$

| A \ BC |    |    |    |    |
|--------|----|----|----|----|
|        | 00 | 01 | 11 | 10 |
| 0      | 1  | 1  |    |    |
| 1      |    |    | 1  |    |

$F =$

# 4-variable k-maps:

| A | B | C | D | Minterm                                                       |
|---|---|---|---|---------------------------------------------------------------|
| 0 | 0 | 0 | 0 | <del>m<sub>0</sub></del> $m_0 = \bar{A}\bar{B}\bar{C}\bar{D}$ |
| 0 | 0 | 0 | 1 | $m_1 = \bar{A}\bar{B}\bar{C}D$                                |
| 0 | 0 | 1 | 0 | $m_2 = \bar{A}\bar{B}C\bar{D}$                                |
| 0 | 0 | 1 | 1 | $m_3 = \bar{A}\bar{B}CD$                                      |
| 0 | 1 | 0 | 0 | $m_4 = \bar{A}B\bar{C}\bar{D}$                                |
| 0 | 1 | 0 | 1 | $m_5 = \bar{A}B\bar{C}D$                                      |
| 0 | 1 | 1 | 0 | $m_6 = \bar{A}BC\bar{D}$                                      |
| 0 | 1 | 1 | 1 | $m_7 = \bar{A}BCD$                                            |
| 1 | 0 | 0 | 0 | $m_8 = A\bar{B}\bar{C}\bar{D}$                                |
| 1 | 0 | 0 | 1 | $m_9 = A\bar{B}\bar{C}D$                                      |
| 1 | 0 | 1 | 0 | $m_{10} = A\bar{B}C\bar{D}$                                   |
| 1 | 0 | 1 | 1 | $m_{11} = A\bar{B}CD$                                         |
| 1 | 1 | 0 | 0 | $m_{12} = AB\bar{C}\bar{D}$                                   |
| 1 | 1 | 0 | 1 | $m_{13} = AB\bar{C}D$                                         |
| 1 | 1 | 1 | 0 | $m_{14} = ABC\bar{D}$                                         |
| 1 | 1 | 1 | 1 | $m_{15} = ABCD$                                               |

| AB \ CD |          |          |          |          |
|---------|----------|----------|----------|----------|
|         | 00       | 01       | 11       | 10       |
| 00      | $m_0$    | $m_1$    | $m_3$    | $m_2$    |
| 01      | $m_4$    | $m_5$    | $m_7$    | $m_6$    |
| 11      | $m_{12}$ | $m_{13}$ | $m_{15}$ | $m_{14}$ |
| 10      | $m_8$    | $m_9$    | $m_{11}$ | $m_{10}$ |

Adjacent squares:

- Next to each other
- Wrap top + bottom
- Wrap left + right
- Four corners

Examples:

| AB \ CD | CD |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      |    |    | 1  | 1  |
| 01      |    | 1  |    |    |
| 11      |    | 1  |    |    |
| 10      | 1  |    | 1  | 1  |

$F =$

| AB \ CD | CD |    |    |    |
|---------|----|----|----|----|
|         | 00 | 01 | 11 | 10 |
| 00      | 1  |    |    | 1  |
| 01      | 1  | 1  |    |    |
| 11      | 1  | 1  |    |    |
| 10      | 1  |    |    | 1  |

$F =$