Maxterms & Product-of-sums Standard Form

Consider "majority" example again:

A	B	C	F	$\bar{F}$	"Maxterms"	"Minterms"
0	0	0	0	1	$M_0 = A+B+C$	$m_0 = \bar{A}\bar{B}\bar{C}$
0	0	1	0	1	$M_1 = A+B+\bar{C}$	$m_1 = \bar{A}\bar{B}C$
0	1	0	0	1	$M_2 = A+\bar{B}+C$	$m_2 = \bar{A}B\bar{C}$
0	1	1	1	0	$M_3 = A+\bar{B}+\bar{C}$	$m_3 = \bar{A}BC$
1	0	0	0	1	$M_4 = \bar{A}+B+C$	$m_4 = A\bar{B}\bar{C}$
1	0	1	1	0	$M_5 = \bar{A}+B+\bar{C}$	$m_5 = A\bar{B}C$
1	1	0	1	0	$M_6 = \bar{A}+\bar{B}+C$	$m_6 = AB\bar{C}$
1	1	1	1	0	$M_7 = \bar{A}+\bar{B}+\bar{C}$	$m_7 = ABC$

Sum of products for $\bar{F}$:

$$\bar{F} = m_0 + m_1 + m_2 + m_4$$
$$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

Take complement to get F:

$$F = \bar{\bar{F}} = \overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}}$$
$$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)$$
$$= M_0 \cdot M_1 \cdot M_2 \cdot M_4$$

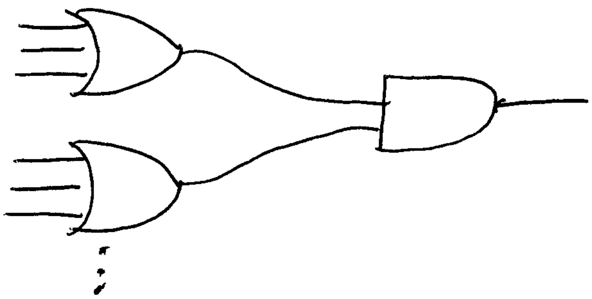
"Product of sums" or "Product of maxterms"

__Note:__

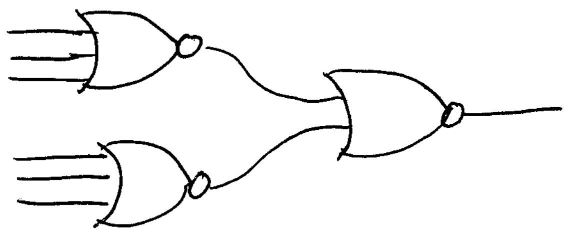
- Each "Factor" corresponds to a 0 in truth table for F
- Each factor equals 0 only for one input combination
- Since factors are ANDed, any __one__ of them equal to 0 makes the result equal to 0; else result is 1.
- Convention for complementing in maxterms is __opposite__ to minterms.
- $M_j = \overline{m_j}$ [Corresponding maxterms & minterms are complements]

Please read in Section 11.5 – analogous to our development of sum-of-products:

- 2-level implementation with OR-AND



- 2-level implementation with <u>NOR</u> only:



Use Prod. of sums for NOR
Use Sum of Prod. for NAND

- NOR is a universal gate:
  Can implement AND, OR, NOT with NOR only, so can implement any Boolean function with NOR

- Can use Boolean algebra to <u>expand</u> any simplified expression into standard product of maxterms form (that is, include missing terms)

<u>Ex</u>: $F = \bar{A}\bar{C} + (A + \bar{B})C$

Recall distributive property: $X + Y \cdot Z = (X + Y)(X + Z)$

$$F = (\underbrace{\bar{A}\bar{C} + A}_{A + \bar{C}} + \bar{B})(\underbrace{\bar{A}\bar{C} + C}_{\bar{A} + C})$$

$$= (A + \bar{B} + \bar{C}) \cdot (\bar{A} + C) \quad = (A + \bar{B} + \bar{C})(\bar{A} + C + \overset{0}{B\bar{B}})$$

$$= (A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C) = M_3 M_4 M_6$$

Can also use K-maps to simplify product-of-sums expressions:

- Place the 0's in the K-map
- Circle groups of 0's as before
- Write the ~~max~~ terms that characterize each group of zeros.
- Take product ~~of sums~~

<u>Ex</u>: Majority

A \ BC	00	01	11	10
0	0	0		0
1	0			

F =