Homework 3 Solution

2.1-2 Signal energies:
(a) $E_x = E_y = 2$
(b) $E_x = E_y = 2\pi$
(c) $E_x = E_y = \pi$

Note $x(t)$ and $y(t)$ are orthogonal in (a) and (b), but not in (c):

$$(a)\quad (x, y) = \int_0^2 x(t)\,y(t)\,dt = 0$$

$$(b)\quad (x, y) = \int_0^{2\pi} x(t)\,y(t)\,dt = 0$$

$$(c)\quad (x, y) = \int_0^{\pi} x(t)\,y(t)\,dt = -\frac{\pi}{2}$$

Note $$E_{x \pm y} = \int_{-\infty}^{\infty} [x(t) \pm y(t)]^2\,dt = E_x + E_y \pm 2\,(x, y)$$

So for (a) + (b), $E_{x \pm y} = E_x + E_y$

For (c), $$E_{x \pm y} = E_x + E_y \pm 2\left(-\frac{\pi}{2}\right) = \begin{cases} \pi, & x+y \\ 3\pi, & x-y. \end{cases}$$

2.1-5 Energy in one period, $-2 \le t \le 2$: $(T = 4)$

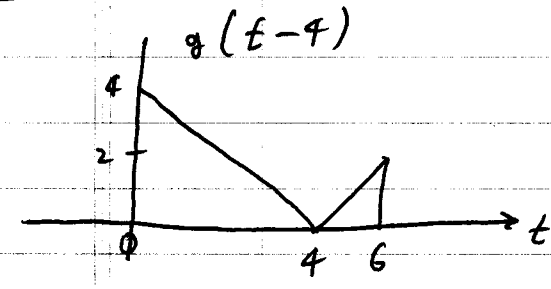
$$\int_{-2}^{+2} t^6\,dt = \frac{1}{7} t^7 \Big|_{-2}^{+2} = \frac{1}{7}\left[2^7 - (-2)^7\right] = \frac{256}{7}$$

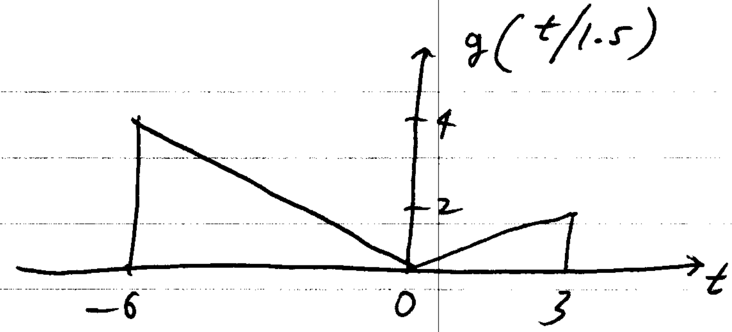
$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)^2\,dt = \frac{64}{7}$$

$$RMS_g = 8/\sqrt{7}$$

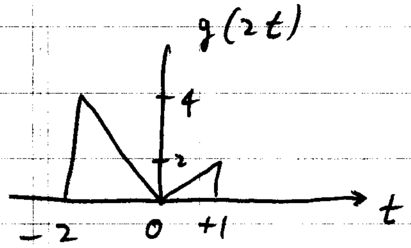
General: $P_{c \cdot g} = c^2 P_g$, $RMS_{c \cdot g} = c \cdot RMS_g$.

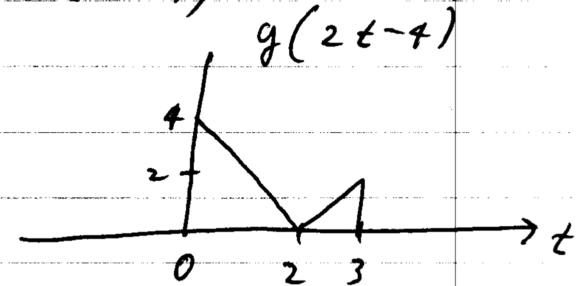
2.3-3



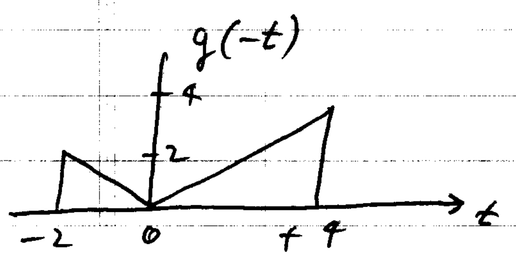


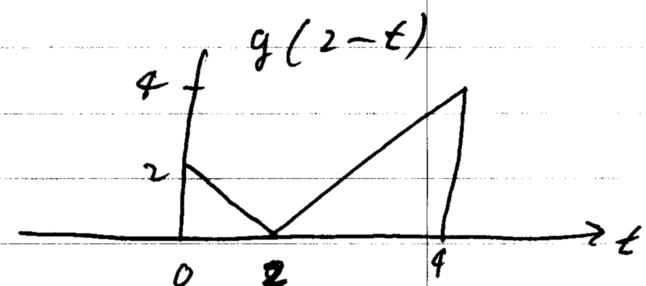
$$g(2t-4) = g\big(2(t-2)\big)$$





$$g(2-t) = g\big(-(t-2)\big)$$





2.4-2

a) $g(\tau)\big|_{\tau=t} = g(t)$

b) $g(t-\tau)\big|_{\tau=0} = g(t)$

c) $e^{-j\omega\tau}\big|_{\tau=0} = 1$

d) $\sin 2\pi$

e) $e^{-t}\big|_{t=-3} = e^{3}$

f) $t^3+4\big|_{t=1} = 5$

g) $g(2-t)\big|_{t=3} = g(-1)$

h) $e^2 \cos\left[\frac{\pi}{2}(3-5)\right]$

$= -e^2$