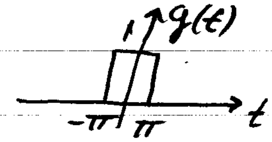
Homework 4 Solution

2.9-1(b)) Period $T_0 = 10\pi$, sketch of one period:



Fundamental frequency $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$

Exponential Fourier series:

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{nt}{5}} = D_0 + D_1 e^{j\frac{t}{5}} + D_{-1} e^{-j\frac{t}{5}} + D_2 e^{j2\frac{t}{5}} + D_{-2} e^{-j2\frac{t}{5}} + \cdots$$

$$D_n = \frac{(g, e^{j\frac{nt}{5}})}{(e^{j\frac{nt}{5}}, e^{j\frac{nt}{5}})} = \frac{1}{10\pi} \int_{-5\pi}^{5\pi} g(t) e^{-jn\frac{t}{5}} dt$$

For $n=0$: $D_0 = \frac{1}{10\pi} \int_{-\pi}^{\pi} 1 \, dt = \frac{2\pi}{10\pi} = \frac{1}{5}$

For $n \neq 0$:

$$D_n = \frac{1}{10\pi} \int_{-\pi}^{\pi} e^{-jn\frac{t}{5}} dt = \frac{1}{10\pi} \left[\frac{1}{-j\frac{n}{5}} e^{-j\frac{nt}{5}} \right]\Bigg|_{-\pi}^{+\pi}$$

$$= \frac{1}{10\pi} \frac{1}{-j\frac{n}{5}} \left[e^{-j\frac{n\pi}{5}} - e^{+j\frac{n\pi}{5}} \right]$$

$$= \frac{2}{10\pi \cdot \frac{n}{5}} \cdot \frac{1}{2j} \left[e^{j\frac{n\pi}{5}} - e^{-j\frac{n\pi}{5}} \right]$$

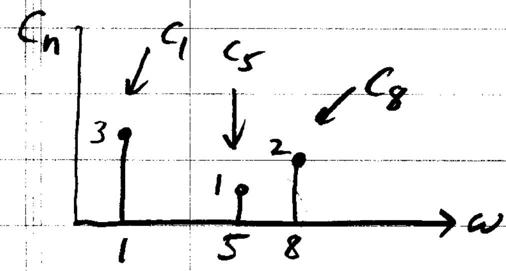
$$= \frac{1}{\pi n} \sin\left(\frac{n\pi}{5}\right)$$

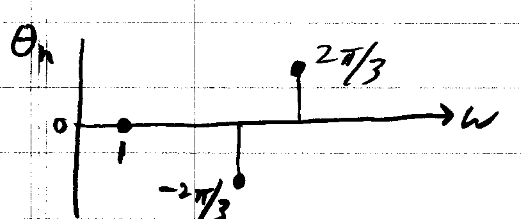
$$\therefore \; g(t) = \frac{1}{5} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \frac{1}{\pi n} \sin\left(\frac{n\pi}{5}\right) e^{j\frac{nt}{5}}$$

See Matlab program for spectra and plot of $g(t)$ using Fourier series.

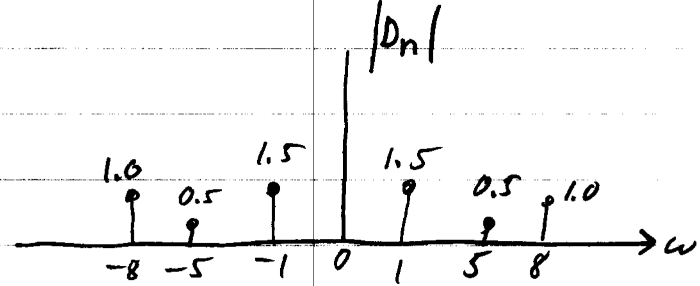
2.9-2 $\quad g(t) = 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t + \frac{2\pi}{3}\right)$

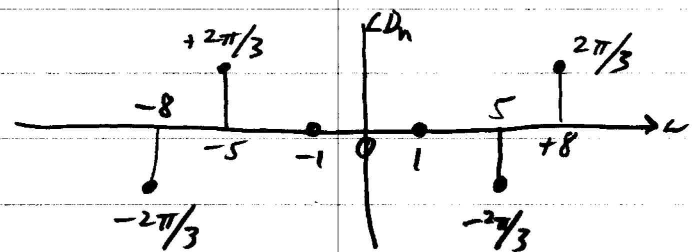
One-sided spectra:





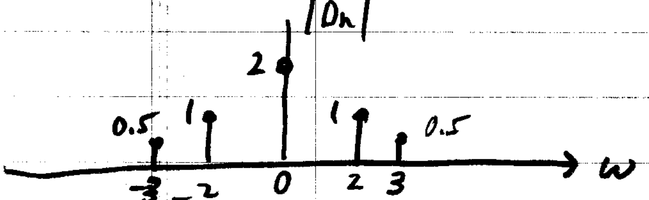
Two-sided spectra:

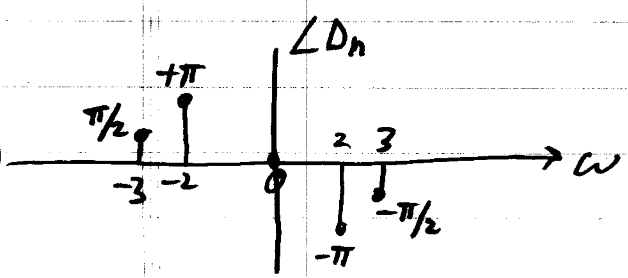




$$g(t) = 1.5e^{jt} + 1.5e^{-jt} + 0.5\, e^{j(5t - 2\pi/3)} + 0.5\, e^{-j(5t - 2\pi/3)}$$
$$+ 1\cdot e^{j(8t + 2\pi/3)} + 1\cdot e^{-j(8t + 2\pi/3)}$$

2.9-3 $\quad g(t) = 2 + 2\cos(2t - \pi) + 1\cdot\cos(3t - \pi/2)$





$$g(t) = 2 + 1\cdot e^{j(2t-\pi)} + 1\cdot e^{-j(2t-\pi)} + 0.5\, e^{j(3t - \pi/2)} + 0.5\, e^{-j(3t-\pi/2)}$$

Same since $\cos\phi = \frac{1}{2}e^{j\phi} + \frac{1}{2}e^{-j\phi}$