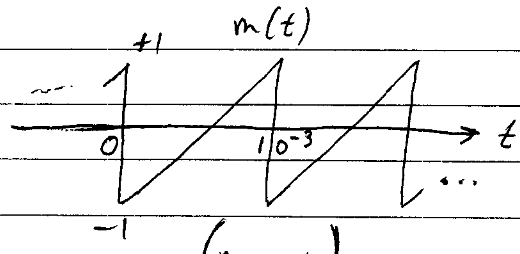
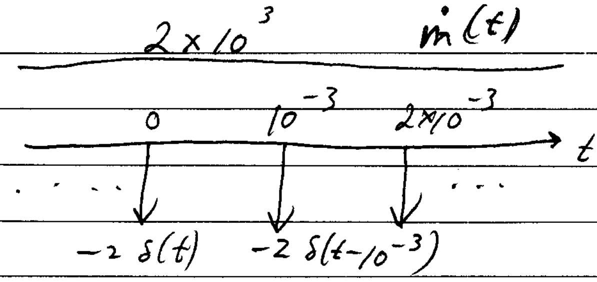
HOMEWORK 9 SOLUTION

(5.1-2)



$(m_p = 1)$



FM: $\omega_i(t) = \omega_c + k_f m(t) \in [\omega_c - k_f m_p,\ \omega_c + k_f m_p]$

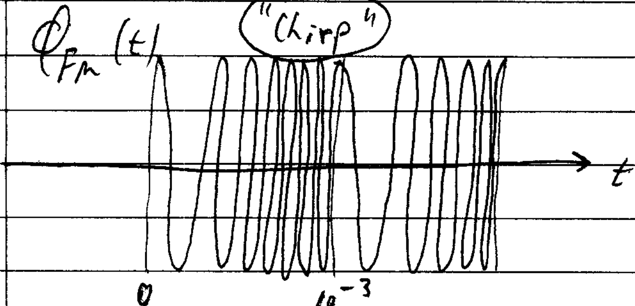
$\in [2\pi(10^6 - 10^3),\ 2\pi(10^6 + 10^3)]$

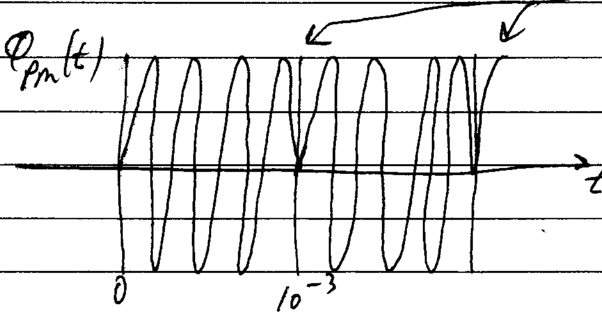
PM: $\omega_i(t) = \omega_c + k_p \dot m(t) = 2\pi(10^6 + 500) - 2\left(\frac{\pi}{2}\right)\sum_{n=-\infty}^{\infty} \delta(t - n(10^{-3}))$

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\,d\alpha = 2\pi(10^6 + 500)t - \pi \sum_{n=-\infty}^{\infty} u(t - n(10^{-3}))$$

$\Rightarrow$ Add $-\pi$ rad to phase every 10^{-3} sec

In general, the phase jump is $2\cdot k_p$ rad, so need $k_p < \pi$ to stay in $[0, 2\pi]$.

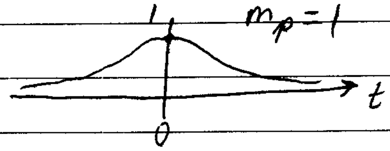


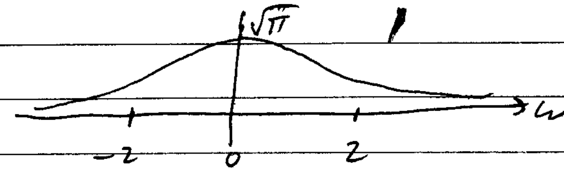


cycles in 10^{-3} sec:

$$\frac{10^{-3}}{(10^6 + 500)^{-1}} = 1000.5$$

(5.2-7) $m(t) = e^{-t^2} \Longleftrightarrow M(\omega) = \sqrt{\pi}\, e^{-(\omega/2)^2}$





message bandwidth $W \approx 2$ rad/sec

$B = W/2\pi = 1/\pi$ Hz

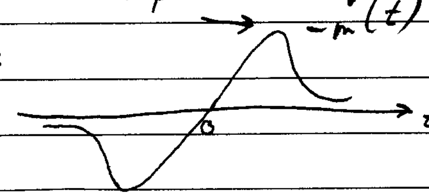
$f_c = 10^4$ Hz, $k_f = 6000\pi$, $k_p = 8000\pi$

a) FM: $\omega_i(t) = \omega_c + k_f m(t) < \omega_c + k_f m_p$

$$\Delta f_{FM} = \frac{k_f m_p}{2\pi} = 3000 \text{ Hz}$$

PM: $\omega_i(t) = \omega_c + k_p \dot{m}(t)$

$\dot{m}_p = \sqrt{2}\, e^{-1/2}$



$$\dot{m}(t) = \frac{d}{dt} e^{-t^2} = -2t\, e^{-t^2}$$

What is peak of $\dot{m}(t)$?

$$\ddot{m}(t) = -2e^{-t^2} + 4t^2 e^{-t^2} = 0$$

$\Rightarrow$ peak at $t = \frac{1}{\sqrt{2}}$

$$\dot{m}\left(\frac{1}{\sqrt{2}}\right) = -\sqrt{2}\, e^{-1/2} = \dot{m}_p$$

$$\Delta f_{PM} = \frac{k_p \dot{m}_p}{2\pi} = 4000\sqrt{2}\, e^{-1/2} = 3431 \text{ Hz}$$

b) Note message $B \ll \Delta f$, so

Carson's rule:

$$B_{FM} \approx \cancel{B} \approx 2(\Delta f_{FM} + B) \approx 2\,\Delta f_{FM} = 6000 \text{ Hz}$$

$$B_{PM} \approx 2\,\Delta f_{PM} \approx 6862 \text{ Hz}$$