

# Entangled States of Three Particles: Greenberger, Horne, and Zeilinger (**GHZ**) states

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## Part I: Application of GHZ states to Quantum Foundations

Entangled states, the EPR paradox, elements of reality, spooky action at a distance, hidden variables, Bell's Theorem, etc.  
(Extension and simplification(?) of material from PHYS 212)

Part II: Understanding a toy GHZ experiment from the interpretive framework of QBism

## Entangled States (Supp. Reading 8.1–8.4)

## Unentangled States (Supp. Reading 8.1–8.4)

Two-particle spin state:

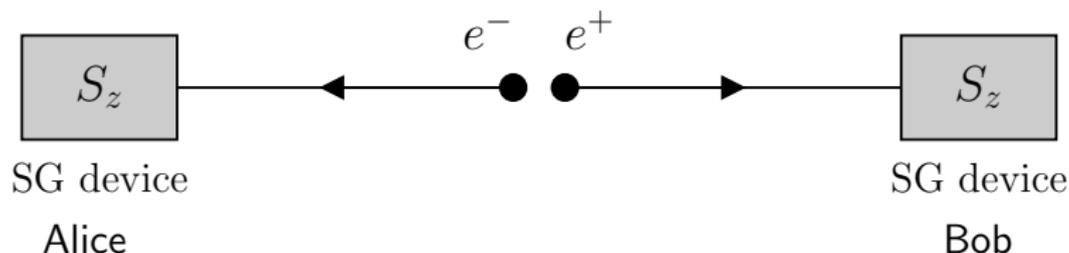
$$|\psi\rangle = \frac{1}{\sqrt{2}} |\uparrow\rangle_A |\uparrow\rangle_B + \frac{1}{\sqrt{2}} |\uparrow\rangle_A |\downarrow\rangle_B$$

This state can be factored:

$$|\psi\rangle = |\uparrow\rangle_A \left( \frac{1}{\sqrt{2}} |\uparrow\rangle_B + \frac{1}{\sqrt{2}} |\downarrow\rangle_B \right)$$

Measurement of particle **A** does not influence state of particle **B**, or any measurement made on particle **B**. States not entangled.

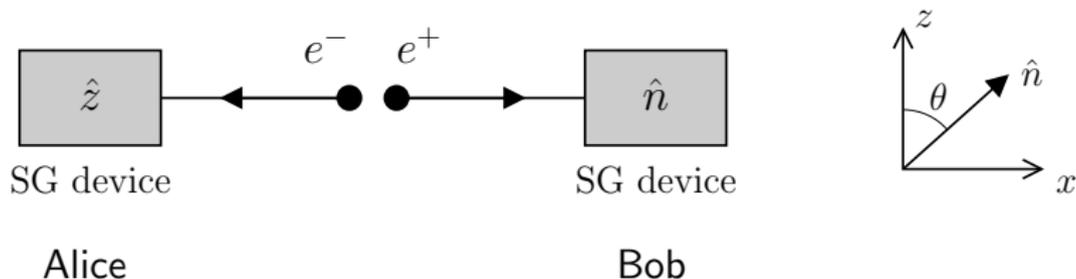
# The EPR thought experiment (Supp. Reading 8.5)



$$\begin{aligned} |\psi\rangle &= \frac{1}{\sqrt{2}} |\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}} |\downarrow\uparrow\rangle \\ &= \frac{1}{\sqrt{2}} (|+z\rangle_A |-z\rangle_B) - \frac{1}{\sqrt{2}} (|-z\rangle_A |+z\rangle_B) \end{aligned}$$

- ▶ State can't be factored, i.e., it's entangled
- ▶ Result of measurement of Alice's spin correlated with value measured that will be measured by Bob
- ▶ "Spooky action at a distance"
- ▶ EPR: Quantum description must not be complete

# Bell's Experiment (Supp. Reading 8.6)



- ▶ Alice measures spin projection of electron along  $\hat{z}$ . Finds it's *down*
- ▶ Bob measures spin projection along  $\hat{n}$ , with  $\theta = 45^\circ$ . What's the probability that Bob measures *spin-up* along  $\hat{n}$ -axis?

# Bell's Theorem (Supp. 8.6)

Quantum Mechanics:

- ▶ Bell spin state:

$$|\psi\rangle = \frac{1}{\sqrt{2}} |\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}} |\downarrow\uparrow\rangle$$

- ▶ Bob measures *spin-up* along  $45^\circ$  axis with probability

$$\text{prob}^{\text{q.m.}} = 85\%$$

Classical *Hidden Variable* Theory:

- ▶ Bell spin state:

$$|\psi\rangle = \frac{1}{\sqrt{2}} |\uparrow\downarrow, ?\rangle - \frac{1}{\sqrt{2}} |\downarrow\uparrow, ?\rangle$$

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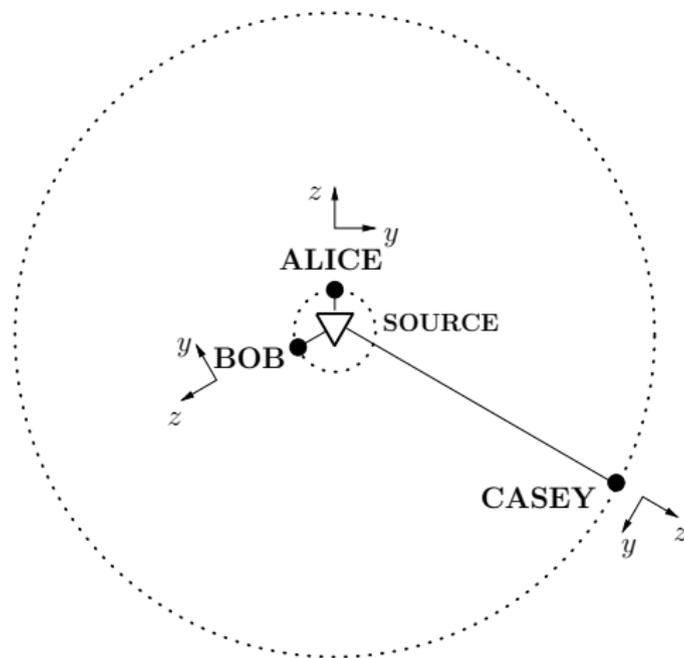
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CONFLICT IS IN PROBABILITIES FOR OUTCOMES

# GHZ Entangled State

Three particles better than two?



# GHZ Entangled State v.1

Extend Bell state to three spin- $\frac{1}{2}$  particles and three observers (Alice, Bob, and Casey):

$$|\psi\rangle_{\text{GHZ}} = \frac{1}{\sqrt{2}} \left( |\uparrow_A \uparrow_B \uparrow_C\rangle \right) - \frac{1}{\sqrt{2}} \left( |\downarrow_A \downarrow_B \downarrow_C\rangle \right)$$

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# GHZ Experiment 1

$$|\psi\rangle_{\text{GHZ}} = \frac{1}{\sqrt{2}} \left( |+\rangle_A |+\rangle_B |+\rangle_C \right) - \frac{1}{\sqrt{2}} \left( |-\rangle_A |-\rangle_B |-\rangle_C \right)$$

- ▶ Alice measures spin projection on  $x$ -axis
- ▶ Bob measures spin projection on  $y$ -axis
- ▶ Casey measures spin projection on  $y$ -axis

## Supp. Chapter 5 to the Rescue

$$|\psi\rangle_{\text{GHZ}} = \frac{1}{\sqrt{2}} \left( |+\rangle_A |+\rangle_B |+\rangle_C \right) - \frac{1}{\sqrt{2}} \left( |-\rangle_A |-\rangle_B |-\rangle_C \right)$$

Transform basis vectors as in Table 5.1 of 212 Supp. Reading:

$$|+\rangle = \sqrt{\frac{1}{2}} |+\rangle_x + \sqrt{\frac{1}{2}} |-\rangle_x$$

$$|-\rangle = \sqrt{\frac{1}{2}} |+\rangle_x - \sqrt{\frac{1}{2}} |-\rangle_x$$

$$|+\rangle = \sqrt{\frac{1}{2}} |+\rangle_y + \sqrt{\frac{1}{2}} |-\rangle_y$$

$$|-\rangle = -i\sqrt{\frac{1}{2}} |+\rangle_y + i\sqrt{\frac{1}{2}} |-\rangle_y$$

## Supp. Chapter 5 to the Rescue

$$|\psi\rangle_{\text{GHZ}} = \frac{1}{\sqrt{2}} \left( |+\rangle_A |+\rangle_B |+\rangle_C \right) - \frac{1}{\sqrt{2}} \left( |-\rangle_A |-\rangle_B |-\rangle_C \right)$$

$$\begin{aligned} |+\rangle_A |+\rangle_B |+\rangle_C &\propto \left( |+\rangle_A + |-\rangle_A \right) \left( |+\rangle_B + |-\rangle_B \right) \\ &\quad \times \left( |+\rangle_C + |-\rangle_C \right) \\ &= |+\rangle_A |+\rangle_B |+\rangle_C + |+\rangle_A |+\rangle_B |-\rangle_C \\ &\quad + \dots \end{aligned}$$

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$$\begin{aligned} |-\rangle_A |-\rangle_B |-\rangle_C &\propto - \left( |+\rangle_A - |-\rangle_A \right) \left( - |+\rangle_B + |-\rangle_B \right) \\ &\quad \times \left( - |+\rangle_C + |-\rangle_C \right) \\ &= - |+\rangle_A |+\rangle_B |+\rangle_C + |+\rangle_A |+\rangle_B |-\rangle_C \end{aligned}$$

$$\begin{aligned} |\psi\rangle_{\text{GHZ}} &= \frac{1}{2} \left( |+x\rangle_A | +y\rangle_B | +y\rangle_C \right) \\ &+ \frac{1}{2} \left( |+x\rangle_A | -y\rangle_B | -y\rangle_C \right) \\ &+ \frac{1}{2} \left( |-x\rangle_A | +y\rangle_B | -y\rangle_C \right) \\ &+ \frac{1}{2} \left( |-x\rangle_A | -y\rangle_B | +y\rangle_C \right) \end{aligned}$$

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- ▶ **Spooky action at a distance:** Like EPR experiment

$$\begin{aligned}
 |\psi\rangle_{\text{GHZ}} &= \frac{1}{2} \left( |+\mathit{x}\rangle_{\text{A}} |+\mathit{y}\rangle_{\text{B}} |+\mathit{y}\rangle_{\text{C}} \right) \\
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 \end{aligned}$$

- ▶ **Spooky action at a distance:** Like EPR experiment
- ▶ **GHZ Rule:** Always find an odd number of *up*'s in an  $S_x S_y S_y$  measurement.  
 (Corollary: If an  $x$  measurement yields *up*, then  $y$  measurements must both yield *up*, or both yield *down*)

## GHZ Experiment 2

$$|\psi\rangle_{\text{GHZ}} = \frac{1}{\sqrt{2}} \left( |+\rangle_A |+\rangle_B |+\rangle_C \right) - \frac{1}{\sqrt{2}} \left( |-\rangle_A |-\rangle_B |-\rangle_C \right)$$

- ▶ Alice measures spin projection on  $x$ -axis
- ▶ Bob measures spin projection on  $x$ -axis
- ▶ Casey measures spin projection on  $x$ -axis

$$\begin{aligned} |\psi\rangle_{\text{GHZ}} &= \frac{1}{2} \left( |+\mathbf{x}\rangle_{\text{A}} |+\mathbf{x}\rangle_{\text{B}} |-\mathbf{x}\rangle_{\text{C}} \right) \\ &+ \frac{1}{2} \left( |+\mathbf{x}\rangle_{\text{A}} |-\mathbf{x}\rangle_{\text{B}} |+\mathbf{x}\rangle_{\text{C}} \right) \\ &+ \frac{1}{2} \left( |-\mathbf{x}\rangle_{\text{A}} |+\mathbf{x}\rangle_{\text{B}} |+\mathbf{x}\rangle_{\text{C}} \right) \\ &+ \frac{1}{2} \left( |-\mathbf{x}\rangle_{\text{A}} |-\mathbf{x}\rangle_{\text{B}} |-\mathbf{x}\rangle_{\text{C}} \right) \end{aligned}$$

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Specific Case of Experiment:

- ▶ If Alice reads *spin-up*, and Bob reads *spin-up*, what will Casey find?

# EPR Realism

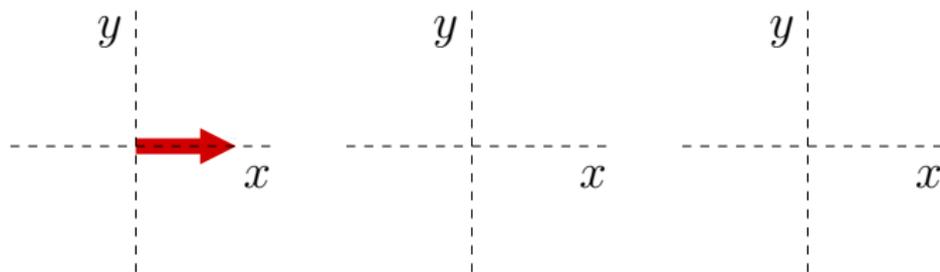
The value of a spin component recorded by Casey satisfies the criteria set by Einstein, Podolsky, and Rosen for an *element of reality*, because Alice,

*“without, in any way disturbing a system [Casey’s particle], can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity [the direction of the projection of Casey’s spin along his x-axis], then there exists an element of reality corresponding to this physical quantity.”*

Einstein, Podolsky, and Rosen, Phys. Rev. (1935)

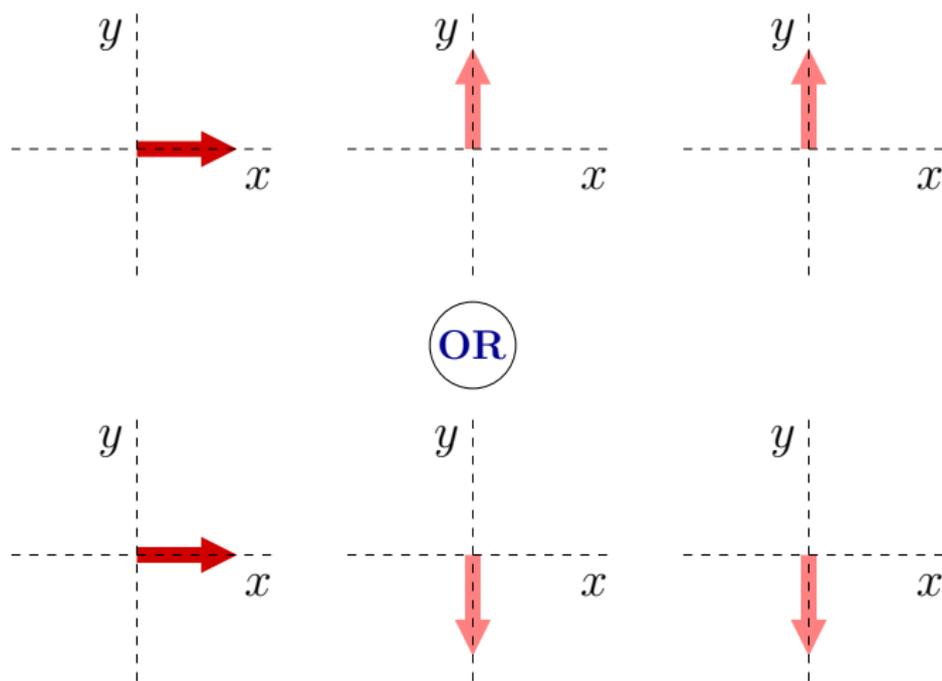
# Experimental Prediction of Realists (Multipart)

## Alice's Observation



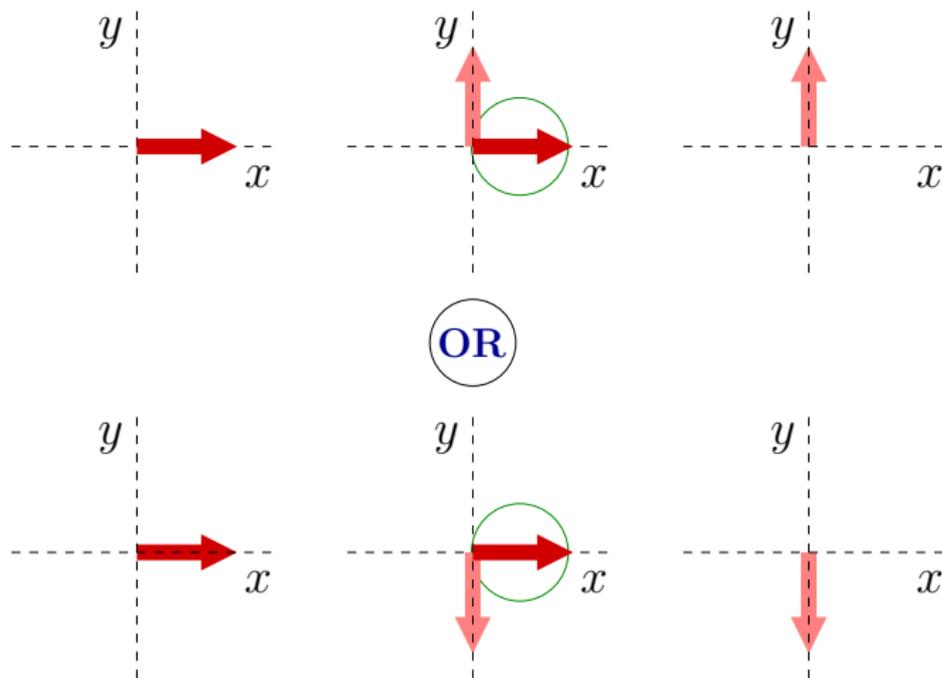
# Experimental Prediction of Realists (using GHZ rule)

## Assignments of Alice's Friend



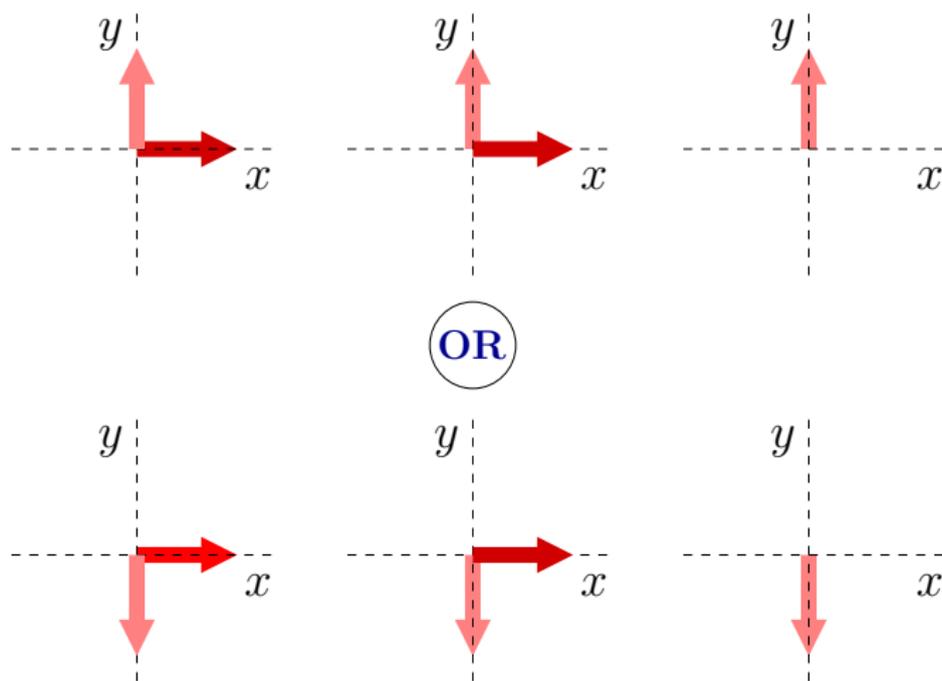
# Experimental Prediction of Realists

## Add Bob's Observation



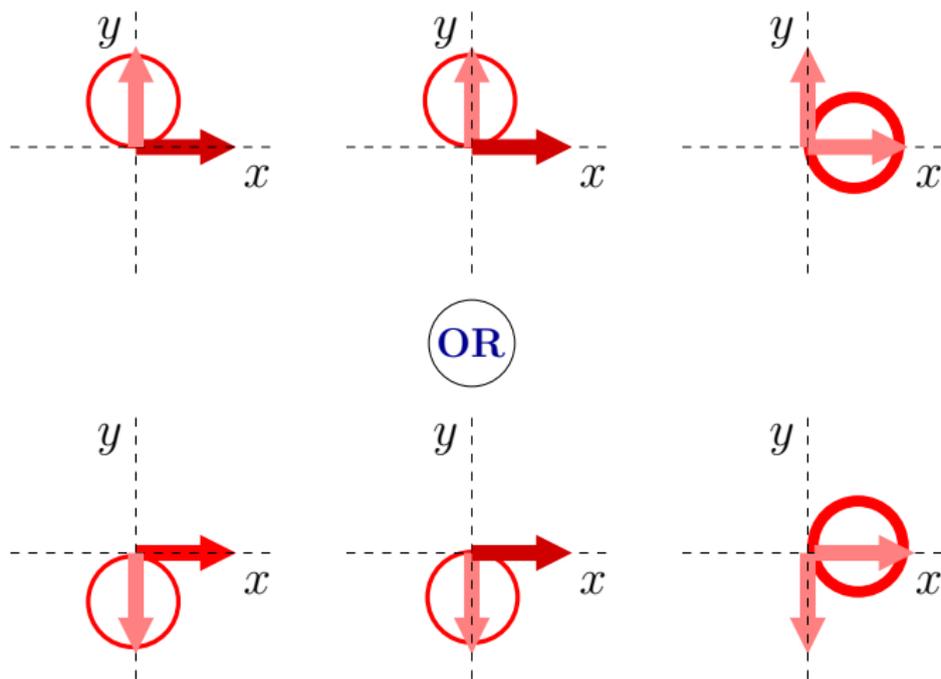
# Experimental Prediction of Realists (Using GHZ Rule)

## Assignments of Bob's Friend



# Experimental Prediction of Realists – FINAL

## After Application of GHZ Rule



Break

## Part 2 of Talk: QBism, a.k.a. Quantum Bayesianism

### BIG Claim:

QBism “removes the paradoxes, conundra, and pseudo-problems that have plagued quantum foundations for the past nine decades”

### Counter-claim:

QBism is “a radical minority view among physicists” that isn't really necessary to resolve foundational issues.

## Summary of QBism – This may not make sense yet!

- ▶ QBism is an interpretation of QM informed by the perspectives of quantum information theory and subjective probability

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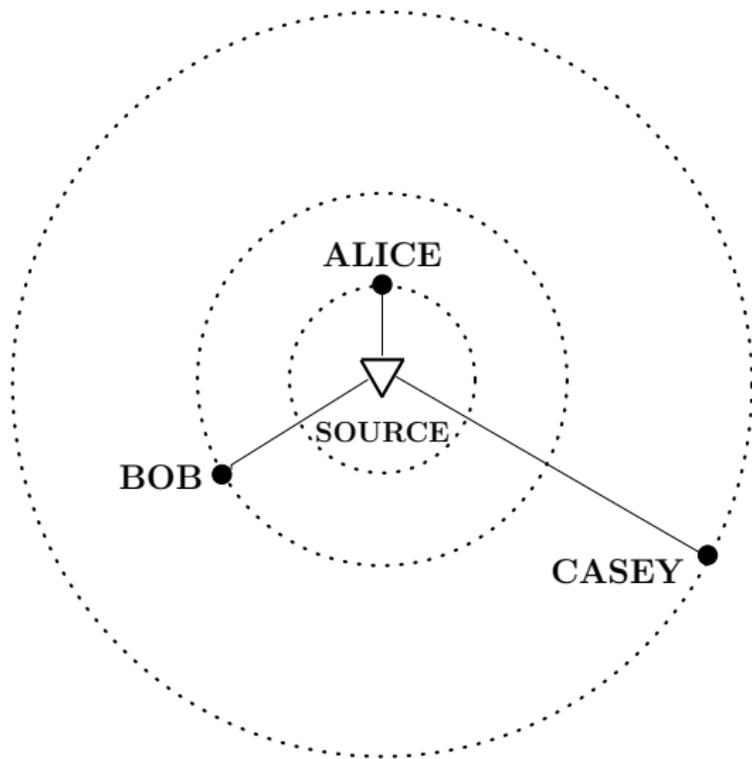
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HERESY? ANTI-SCIENTIFIC?

# Slightly modified geometry for GHZ experiment



# GHZ Rule and EPR revisited

Reminder: “Useful” form for  $|\psi\rangle_{\text{GHZ}}$  for  $x$ - $y$ - $y$  measurement is

$$\begin{aligned} |\psi\rangle_{\text{GHZ}} &= \frac{1}{2} \left( |+\!x\rangle_{\text{A}} |+\!y\rangle_{\text{B}} |+\!y\rangle_{\text{C}} \right) \\ &+ \frac{1}{2} \left( |+\!x\rangle_{\text{A}} |-\!y\rangle_{\text{B}} |-\!y\rangle_{\text{C}} \right) \\ &+ \frac{1}{2} \left( |-\!x\rangle_{\text{A}} |+\!y\rangle_{\text{B}} |-\!y\rangle_{\text{C}} \right) \\ &+ \frac{1}{2} \left( |-\!x\rangle_{\text{A}} |-\!y\rangle_{\text{B}} |+\!y\rangle_{\text{C}} \right) \end{aligned}$$

**Question:** Alice, Bob, and Casey make measurements at widely separated times. What about “wavefunction collapse” in this context?

Alice's initial *assignment* of state vector:

$$\begin{aligned} |\psi_0\rangle_A &= \frac{1}{2} \left( |+\!x\rangle_A |+\!y\rangle_B |+\!y\rangle_C \right) \\ &+ \frac{1}{2} \left( |+\!x\rangle_A |-\!y\rangle_B |-\!y\rangle_C \right) \\ &+ \frac{1}{2} \left( |-\!x\rangle_A |+\!y\rangle_B |-\!y\rangle_C \right) \\ &+ \frac{1}{2} \left( |-\!x\rangle_A |-\!y\rangle_B |+\!y\rangle_C \right) \end{aligned}$$

- ▶ Alice willing to bet on outcome of the three measurements of Alice, Bob, and Casey.
- ▶ Example: she's willing to bet that the outcome will be (*down, down, up*)
- ▶ She will pay \$0.25 for a ticket that she can redeem for \$1.00 if the result is, in fact, (*down,down,up*)

## Alice reads *spin-up*

- ▶ Alice loses her bet on (*down, down, up*) — but it wasn't a bad bet
- ▶ Alice updates her state vector:

$$|\psi_1\rangle_A = \frac{1}{\sqrt{2}} |+\!x\rangle_A \left( |+\!y\rangle_B |+\!y\rangle_C + |-\!y\rangle_B |-\!y\rangle_C \right)$$

- ▶ Alice now knows that Bob and Casey will agree; either both will read *spin-up* or both read *spin-down*
- ▶ Alice now willing to buy a ticket for \$0.50 that will pay \$1.00 if Bob and Casey both read *spin-up*

## Bob reads *spin-up* in his distant lab

- ▶ What changes for Alice? **NOTHING!**
- ▶ Alice's bet is still consistent with her experience; she won't lose money by betting based on her assigned state vector  $|\psi_1\rangle_A$ .
- ▶ No call for a concept like *wavefunction collapse*.
- ▶ (Bob will update his state vector based on the local information available to him, but right now I'm following the thread of Alice as agent).

Alice receives word of Bob's reading of *spin-up* from his distant lab

- ▶ The results of Bob's experiment have now entered Alice's experience
- ▶ Alice updates her state vector:

$$|\psi_2\rangle_A = | +x \rangle_A | +y \rangle_B | +y \rangle_C$$

- ▶ Alice predicts with certainty that the result she will eventually hear from Casey is *spin-up*

# Summary of QBism

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## GHZ States:

- ▶ Greenberger, Horne, Shimony, and Zeilinger, "Bell's theorem without inequalities," *Am. J. Phys.*, **58** 1131 (1990)
- ▶ Mermin, "Quantum mysteries revisited," *Am. J. Phys.*, **58**, 731 (1990)
- ▶ Pan, et al., "Experimental test of quantum nonlocality in three-photon GHZ entanglement," *Nature* **403** 515 (2000)

## QBism:

- ▶ Mermin, "Commentary: Fixing the shifty split," *Phys. Today*, **65**, 8 (2012), and responses in *Phys. Today*
- ▶ von Baeyer, *QBism, The Future of Quantum Mechanics* (Harvard University Press, Cambridge, MA 2016)\*
- ▶ Fuchs, Mermin, and Schack, "An introduction to QBism . . .," *Am. J. Phys.* **82**, 749 (2014)
- ▶ Fuchs and Schack, "Quantum-Bayesian coherence," *Rev. Mod. Phys.* **85** 1693 (2013)

## Path to this talk & thank yous

- ▶ David Mermin's 1990 article on the GHZ states in AJP alerted me to the fact that there was something more than Bell's Theorem (but I was an untenured assistant professor at the time). [Danny Greenberger had an office down the hall from mine when I was teaching at CCNY.]
- ▶ David Mermin's 2012 Commentary in Physics Today made me think that there was something interesting to think about in QBism (but I was department chair at the time, without time to concentrate on such things).
- ▶ Part of Hans Christian von Baeyer's popular science book on QBism helped me connect GHZ states to QBism. He also provided valuable encouragement on a manuscript.
- ▶ Blake Stacey, of the Physics Department at UMass Boston, provided extremely valuable feedback on my manuscript from the point of view of a committed and expert QBist