PHYS 331 — Problem Set #14

Reading for the rest of the semeseter: Chapter 16

Problems to be handed in Wednesday 12/4:

1. Consider a rigid body rotating about its center of mass.

   (a) Use the Einstein sum convention, along with the Levi-Civita and Kronecker-delta symbols, to show that the kinetic energy of a point mass $m^{(\alpha)}$, with displacement from the center of mass $\mathbf{r}^{(\alpha)}$, can be written in the form

   $$T^{(\alpha)} = \frac{1}{2} m^{(\alpha)} \omega_j \left[{r^{(\alpha)}}^2 \delta_{jk} - r_j r_k \right] \omega_k.$$

   There is one little trick that comes in handy at one point here. If you represent a vector $\mathbf{u}$ as a column matrix, then u^2 (the dot product of $\mathbf{u}$ with itself) can be represented as $\mathbf{u}\mathbb{1}\mathbf{u}$, where $\mathbb{1}$ is the identity matrix.

   (b) Use the result above to show that the kinetic energy of the rigid body can be written as in Taylor's Eq. (10.67):

   $$T = \frac{1}{2} \boldsymbol{\omega} \cdot \mathbf{L} = \frac{1}{2} \tilde{\boldsymbol{\omega}} \mathbf{I} \boldsymbol{\omega}.$$

   You should then take a look at the method proposed by Taylor in problem 10.33. (You do not have to complete the proof as suggested in the problem.)

2. Taylor 16.20 — express your answer in terms of the product $dx\, dy$.

3. An elastic medium has a stress tensor as a function of position given by

   $$\mathbf{\Sigma} = \begin{pmatrix} -2+x & z^2 & yz \\ z^2 & -2+y & y^2 \\ yz & y^2 & -2+z \end{pmatrix}$$

   For each of the cases below, find the force on a surface of area dA at the specified location and orientation. Express your answeress in terms if the appropriate infinitesimal elemens dx, dy, and dz.

   (a) At the point $(1, 1, 0)$ and oriented in the $+\hat{x}$ direction.

(b) At the point $(1, 1, 0)$ and oriented in the $+\hat{z}$ direction.

(c) At the point $(0, 1, 0)$ and oriented in the $+\hat{y}$ direction.

(d) At the point $(0, 0, 1)$ and oriented so that $\hat{\mathbf{n}} = \frac{1}{\sqrt{2}}(\hat{x} + \hat{z})$.

(e) For each of (a)–(d) above, state whether there is a non-zero shear force or not.

4. In this problem you will use a "sledge hammer" to calculate the buoyant force on a cube with sides of length a submerged in a fluid in hydrostatic equilibrium.

   (a) In class we showed that the pressure p in a fluid is

   $$p(z) = p_0 + \rho g z,$$

   where p_0 is the pressure at the surface of the fluid, and z is the distance below the surface (z is positive down). Write an expression for the stress tensor in the fluid.

   (b) Use your stress tensor to calculate the net surface force on a cube with sides of length a that that is completely submerged in the fluid. (The edges of the cube are aligned with the conventional x-, y-, and z-axes.)