Approach for E-field Integrals

1. Draw a sketch, choose integration variable (x, y, or θ)

2. Pick a tiny piece of charge and label it dq

   a. label the size of this tiny piece using dx, dy, or $R\,d\theta$

   b. Draw r (distance between dq and P) on the sketch

   c. Draw an arrow for $d\vec{E}$ at P due to dq

3. Find dE magnitude in terms of integration variable:

   a. Find dq. Line: $dq = \lambda\,dx$ or $dq = \lambda\,dy$. Arc: $dq = \lambda\,Rd\theta$

   b. Find r (use Pythagoras)

   c. Plug dq and r into $dE = k\,dq/r^2$

4. Determine the components $dE_x = \pm dE\,\cos\theta$ and $dE_y = \pm dE\,\sin\theta$. You may need to use similar triangles. Use your drawing to determine the signs.

5. Determine the limits of integration (where is the charge?)

6. Put it together and solve for $E_x = \displaystyle\int dE_x$ and $E_y = \displaystyle\int dE_y$.

Useful Integrals

$$\int \frac{x\,dx}{(a^2+x^2)^{3/2}} = \frac{-1}{\sqrt{a^2+x^2}}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

$$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax)$$

$$\int e^{-bx}\,dx = -\frac{1}{b}e^{-bx}$$

$$\int xe^{-bx}\,dx = -\left(\frac{x}{b} + \frac{1}{b^2}\right)e^{-bx}$$

$$\int x^2e^{-bx}\,dx = -\left(\frac{x^2}{b} + \frac{2x}{b^2} + \frac{2}{b^3}\right)e^{-bx}$$

$$\int x\sin^2(ax)\,dx = \frac{x^2}{4} - \frac{x}{4a}\sin(2ax) - \frac{1}{8a^2}\cos(2ax)$$