Lab 20

Radioactive Decay

Continuing Objectives

3. Be able to write an experimental result (including correct number of significant digits, uncertainty, units).

5. Know how to keep a clear and organized record, including an introduction (with purpose of lab and appropriate laws or equations), apparatus sketch, table of raw data and calculated quantities, and a good conclusion or summary.

6. Be able to make a good graph, either in your notebook or with a computer, including labels, scales, units, dependent, and independent variables.

7. Know how to make comparisons: are two measured quantities equal? Is a measured quantity statistically equivalent to a theoretical value?

Introduction

Many atoms and nuclei are energetically unstable and thus *radioactive*. Radioactive nuclei decay into more stable particles with the release of energy in the form of kinetic energy, photons, or other particles. The number of these unstable particles that decays in a given time interval is not constant, but rather is proportional to the amount of particles that are present. The time for half of an initial number of radioactive nuclei of a particular to decay is called the *half-life*. Because each type of radioactive nuclei (also called a radionuclide) has a unique half-life, determining the half-life of an unknown radionuclide is an important method of identification.

As a consequence of the rate of decay being proportional to the number of particles that are present, the number of radionuclides present in any given sample will decrease exponentially. This result can be shown mathematically. The rate of decay changes with time: the more sample there is, the more will decay, and the less sample there is, the less will decay. In radioactive processes, this kind of exponential decay occurs because the decay process is governed by quantum mechanics, and quantum mechanics is inherently probabilistic. For any given nucleus, we can only predict the probability for it to decay in a given time interval, and can never say for certain when a given nucleus will decay. You might think that because of this randomness, we can't say anything at all about the decay of a large collection of nuclei (after all, each individual nucleus obeys the laws of quantum mechanics), but in fact we can.

In Part I of this experiment we will simulate the decay of a nucleus by using the roll of dice. Since the roll of a die is treated as random, we can simulate quantum behavior. You will roll a box of dice at your lab bench and record the results for your individual group. Your entire lab group will also collect their results, and you will begin to see how, even though the roll of a single die is random, the behavior of a large number of dice begins to follow a very predictable pattern. For Part II of this lab, you will measure the half-life of radioactive ^{137}Ba. Metastable ^{137}Ba nuclei have a half-life of a few minutes, a time which is easily measured in the laboratory.

Part I: Simulating Radioactive Decay

In this part of the lab, you will simulate radioactive decay using dice. We will treat each die as a metastable nucleus (a nucleus that will decay). Since according to quantum mechanics, the only thing we know is the probability that the nucleus will decay, we simulate that by rolling the die: if the die comes up a 3, we will say that nucleus has decayed. (Note that this choice is arbitrary; we could have chosen any number to represent a decayed nucleus.)

1. At your lab bench there is a plastic box full of dice, two of which are colored differently from the rest. Each die represents a metastable nucleus, whose state (decayed or not) as time passes is determined by rolling the dice at fixed intervals (each of which represents a jump in time). If a die returns 3, the nucleus has decayed. Suppose that you focus on just one die; you would like to predict when it decays. Predict how many rolls (i.e. time intervals) it will take before it decays. Take one of the differently colored dice and roll it six times. Does it decay during this set of intervals? Repeat this twice. Can you predict when the die will decay?

2. You and your partner should each roll your differently colored die, and see whose simulated nucleus "decays" first. How many rolls did it take for your nucleus to decay? How many rolls did it take for your partner's to decay?

   Since the decay of the nucleus is random, it seems like we can't make any kind of useful prediction. However, since the decay is probabilistic (and we assume that each identical nucleus has the same probability of decay), we can make useful predictions for large numbers of nuclei.

3. Put your differently colored dice back in the box. Each box should be labeled with the number of dice it contains. To ensure that the number is accurate, first recount the number of dice in the box. Hold the lid tightly on the box, and shake it thoroughly. Carefully open the lid, and remove from the container any nuclei which have decayed (i.e., turned up as a '3'). In Excel record the number of dice you have removed from the container. Since you know how many dice you started with, you should be able to calculate how many dice remain in the box.

4. Repeat this process of shaking and removing 12 times (for a total of 13 shakes) and record your data. Once you have removed dice from the box, they should not be put back in; in other words, the number of dice in the box should be decreasing with each shake. You should make a table in your lab notebook that includes the number of shakes completed (starting with 0) and the number of nuclei that remain (for each number of completed shakes). You should have noticed that the number of nuclei which decayed in each successive shaking interval (the rate of decay) decreased as the number of nuclei decreased. Did the nuclei collectively behave in some regular fashion? You'll explore this in the following steps. Also, note if and when your differently colored nucleus (from earlier) decayed.

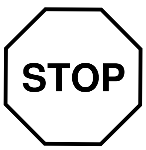
   QUESTION: Could you have predicted when that particular colored nucleus would have decayed? Record your response to these questions in your notebook.

5. Plot a graph of number of remaining (undecayed) nuclei vs. shake number (which is like time). You should also include the initial number of nuclei in the box at 0 shakes (or $t = 0\,\mathrm{s}$). Add gridlines to the graph: click on the + next to the top right corner of the graph $\longrightarrow$ Gridlines: click each option active.

6. Use your data and your graph to estimate the number of shakes (or amount of time) it took for the initial number of nuclei to reduce by half. Print out your plot and paste it into your notebook. On your plot indicate with a horizontal line half of the initial number of nuclei and indicate with a vertical line when this occurs. This also gives you an estimate for the number of shakes (or time) it took for the number of nuclei to drop to half of the value you just indicated (in other words, how many shakes or how much time to drop from half the initial nuclei number to one quarter the initial nuclei number). Indicate this time (or shake number) on your plot. Finally, estimate the number of shakes (or time) it took for the number of nuclei to drop to half of that value (in other words, the number of shakes or the amount of time it took for the number of nuclei to go from one quarter to one eighth the initial number). You have just been estimating the *half-life* of your simulated radioactive decay. Were these various estimates of the half-life close in value to each other?

   Even though you rolled many dice at your lab bench, it is possible that you do not have enough dice to get adequate statistics. For example, if the probability of rolling a 3 on a fair six-sided die is 1/6, you would have expected 1/6th of your atoms to decay in each shaking interval. So even though you could not predict **which particular** nuclei would decay, you could safely guess that 1/6th of the ones that were there would decay in any particular interval.

7. However, as mentioned above, you may not have had enough dice to get adequate statistics. Rather than taking even more data, you will utilize the data gathered by your enitre lab section. Copy your data over to the `Dice Counts` Google Doc, found in the `PHYS 211_212 Lab` folder. Once everyone in your section has finished taking data, tally up the total number of nuclei everyone started with, and the number of nuclei remaining at the beginning of each shake. Make another graph similar to the graph you made in Step 5, and repeat Step 6.



Show your graphs and estimates of half-lives to your instructor or TA.

Part II: Measuring Radioactive Decay

In this part of the lab, you will use a Geiger-Muller detector connected to a computer through a LabPro interface to measure the rate of gamma ray emission of metastable ^{137}Ba nuclei. Using the changing gamma ray emission rate, you will determine the half-life of this radioactive substance.

The ^{137}Ba nuclei are produced in an excited, or metastable, state as a result of the radioactive decay of ^{137}Cs, which has a half-life of about 30 years. The decay scheme is in two parts: the ^{137}Cs decays to produce ^{137}Ba in a metastable excited state (Ba*), and then the excited ^{137}Ba decays to the stable ground state of barium.

$$\begin{aligned} {}^{137}\text{Cs} &\longrightarrow {}^{137}\text{Ba}^* + e^- + \bar{\nu}_e \\ {}^{137}\text{Ba}^* &\longrightarrow {}^{137}\text{Ba} + \gamma \end{aligned} \tag{20.1}$$

where the * symbol indicates a metastable state, e^- indicates an electron, $\bar{\nu}_e$ indicates an elementary particle called an anti-neutrino, and γ indicates a gamma ray.

Each ^{137}Ba* nucleus in this experiment decays by emitting a gamma ray γ (or high energy photon) of energy 662 keV, leaving the nucleus in its stable, nonradioactive ground state. Though we cannot directly count the number of ^{137}Ba* nuclei present at any time, we can count the gamma rays that are emitted. Because the number of these gamma rays detected per unit time is directly proportional to the number of radioactive nuclei in the sample, we can infer the decrease in the number of ^{137}Ba* nuclei by the decrease in the gamma ray detection rate.

A Geiger-Muller (GM) detector consists of a tube of gas held between a fixed potential difference. A gamma ray photon entering the tube can ionize many gas atoms. The ions are accelerated through the potential difference, resulting in a current pulse which is electronically amplified and counted.

Theory

The net rate of change of radioactive ^{137}Ba* nuclei is equal to their rate of production minus their rate of decay. Here, because the half-life of ^{137}Cs is so long, the production rate for ^{137}Ba* is nearly constant. The decay rate is proportional to the number, N, of ^{137}Ba* nuclei present at any instant. In the sample you will obtain for this experiment, the production rate of the ^{137}Ba* will be zero because there will no longer be any ^{137}Cs nuclei around to produce them (you'll see why this is later on in the Procedure section).

In the case where the production rate of ^{137}Ba* is zero, the differential equation which governs the rate of change of the amount of ^{137}Ba* is

$$\frac{dN}{dt} = -BN. \tag{20.2}$$

So we can see that dN/dt (the rate of change of the ^{137}Ba*) is both proportional to N (the amount of ^{137}Ba* in the sample) and negative, which means the amount of ^{137}Ba* in the sample is decreasing. The solution to the differential equation (20.2) is

$$N = N_0 e^{-Bt} = N_0 e^{-\left(\frac{\ln 2}{T}\right)t}, \tag{20.3}$$

where N_0 is the number of nuclei at time $t = 0$, and T is the half-life of the radioactive material being studied. The constant $B = \ln 2/T$ is called the decay constant.

Exercise: Insert the expression for N given in Eq. (20.3) into Eq. (20.2). Simplify the result to show that both sides of the equation are equal. This verifies that the function given in Eq. (20.3) is a solution to the differential equation in Eq. (20.2). So now we have a mathematical relation that verifies our idea that radioactive decay is indeed an exponential decay.

Exercise: Use Eq. (20.3) to find the number of particles left when the time t is equal to the half-life T. Express your answer in terms of N_0. Also, find the number of particles when the time t is equal to twice the half-life. Use your answers to explain why we call the time T the half-life.

In our simulation of radioactive decay in Part I, we were able to directly count the number of dice which "decayed." However, we don't have a way to directly count the number of ^{137}Ba* that either remain or have decayed during one counting interval. However, it is possible to detect the gamma ray photons which are emitted during the decay. The rate of detection of the photons, $R(t)$, is proportional to the rate at which nuclei decay, dN/dt (it is only proportional, because not all of the gamma rays are detected). Furthermore the rate of decay, dN/dt, is proportional to the number of radioactive nuclei present, so we have

$$R(t) = R_0 e^{-Bt} = R_0 e^{-\left(\frac{\ln 2}{T}\right)t}, \tag{20.4}$$

where R_0 is the initial detection rate.

The best way to find an accurate value for the half-life is to plot $\ln(R)$ vs. t. Taking the natural log of both sides of Eq. (20.4) gives

$$\ln(R) = \ln(R_0) - \left(\frac{\ln 2}{T}\right) t. \tag{20.5}$$

Therefore, a graph of $\ln(R)$ vs. t should be a straight line with slope $\left(-\frac{\ln 2}{T}\right)$.

Procedure

Read through this entire section before beginning to count gamma rays emitted by your sample.

We want to observe as much of the decay as possible, so we need to make sure that we are completely set up before we begin to count pulses produced by the gamma ray photons from ^{137}Ba*. Make sure that you understand how to take all of the data you will need before obtaining a source from your instructor or TA.

Begin by turning on your LabPro interface, which will power the Radiation Monitor on your desk. The Radiation Monitor houses the GM detector. Open the file `radioactiveDecay.cmbl` in the `PHYS 211_212 Lab` folder. The Radiation Monitor will sum up the total number of counts per ten seconds and display them in a table and on a graph in LoggerPro. The bottom of the Radiation Monitor should be about 2 to 2.5 cm from the table or base.

Press the green `Collect` button and let the Radiation Monitor collect data for about 5 intervals (so 50 seconds) without a source. Do you get any counts? Where do they come from? Either before or after you take readings from your ^{137}Ba* sample you will have to measure the average number of these background counts during a 10-second interval. This average background number must be subtracted from each count of your sample. You will be collecting data for 500 s, so how long should you collect background counts in order to get an accurate measurement? (*Note: the 50 seconds of data you took so far is not sufficient!*) Ask your instructor or TA if you are not sure.

The TA or instructor will provide you with ^{137}Ba* (already extracted from the ^{137}Cs. [1]

When you receive your sample, place it directly under the detector and immediately start recording the counts in successive 10-second intervals. It is important that the source is centered underneath the tube of the detector so that you get the maximum number of counts from the decay. Logger Pro will display the data in a table and a graph. As the data comes in, make sure it looks like what you expect. Continue for about 50 intervals (500 sec). Then immediately pour your sample into the waste bottle provided.

Analysis

1. Use Excel to make a table with column entries for time, total count rate, count rate minus the average background (R), and $\ln(R)$. In Excel, the command for $\ln(R)$ is `LN(R)`.

2. Make a graph of R vs. time. Add gridlines to your graph and print the graph. Does this graph look exponential? Estimate the half-life of your sample by finding the time for the rate to drop to 1/2 its initial value. Then find the time for the rate to drop to 1/2 of that value. Then find the time for the rate to drop to 1/2 of that value. You should get 3 different estimates of the half-life. Are they consistent with each other?

   Note how remarkable these results are. Even though it is *impossible* to predict when any given ^{137}Ba* will decay (because of the quantum mechanical nature of the decay), we can still see that in an amount of time equal to the half-life, one half of the ^{137}Ba* present in the sample will decay. In the next half-life, one half of the remaining ^{137}Ba* will decay (and so on). So even though it is still impossible to predict when any given nucleus will decay, we have an excellent prediction about a collection of nuclei.

3. To obtain a more precise value of the half-life, make a graph of $\ln(R)$ vs. time (don't print it out yet). This graph will probably appear linear. Add a best-fit line to your data, by first right-clicking once on the data in your chart to select them and then selecting `Add Trendline`. To display your equation on

[1] The ^{137}Cs is contained in a small plastic cylinder. To obtain a small sample of ^{137}Ba* without any ^{137}Cs in it, about 5 ml of 1/40 molar hydrochloric acid will be pushed through the cylinder, and the eluant will be collected in a small container. Barium is soluble in the dilute acid but cesium is not, so this method effectively extracts the ^{137}Ba* while leaving the ^{137}Cs in the plastic cylinder.

the graph, select `Display Equation` from the `Format Trendline` menu that appears on the right side of the screen.

4. Determine half-life T. Recall from Eq. 20.5 that

$$\text{slope} = -\frac{\ln 2}{\mathrm{T}}. \tag{20.6}$$

5. To determine the uncertainty in the half-life we will collect our lab group values for T. Report in scientific notation your result for the half-life of $^{137}\mathrm{Ba}^{*}$. Compare your result with the estimated values that you found in Step 2 and also compare with the accepted value of 153.1 s.

6. **Write one last conclusion.** Specifically, you should include (a) a statement whether radioactive decay is well described by Eq. (20.4) and (b) report the measured half-life of $^{137}\mathrm{Ba}^{*}$ (including uncertainty) and the comparison with the accepted value.