Measurement of the dispersion relation of capillary waves by laser diffraction

Feng Zhu, Runcai Miao, Chunlong Xu, and Zanzan Cao

Citation: American Journal of Physics **75**, 896 (2007); doi: 10.1119/1.2750379
View online: https://doi.org/10.1119/1.2750379
View Table of Contents: https://aapt.scitation.org/toc/ajp/75/10
Published by the American Association of Physics Teachers

Measurement of the dispersion relation of capillary waves by laser diffraction

Feng Zhu[a)]
Institute of Physics and Information Technology, Shaanxi Normal University, Xi'an 710062, China and Department of Mathematics and Physics, Xi'an Communication Institute, Xi'an 710106, China

Runcai Miao
Institute of Physics and Information Technology, Shaanxi Normal University, Xi'an 710062, China and Department of Physics, Yulin College, Shaanxi Yulin 719000, China

Chunlong Xu
Institute of Physics and Information Technology, Shaanxi Normal University, Xi'an 710062, China

Zanzan Cao
Sixth Medicine, Xi'an Railway Hospital, Xi'an 710004, China

(Received 22 January 2007; accepted 25 May 2007)

We describe a simple method for generating capillary waves of a known frequency on a liquid surface. A noncontact technique based on laser diffraction is introduced to measure the wavelength of the capillary waves with great precision. The data give the dispersion relation of capillary waves and provide an accurate method for determining the relation between the surface tension and the temperature in a nondestructive way.
[DOI: 10.1119/1.2750379]

I. INTRODUCTION

Recently many techniques have been used to investigate the characteristics of liquid surface waves, giving a large amount of information about the liquid surface.[1–5] Surface waves on fluids with wavelengths of a few millimeters and less are well known capillary waves. Typically the amplitude of these waves is of the order of one micrometer. In this regime the dominant restoring force is the surface tension.[6–8] Weisbuch has considered capillary waves as a reflection diffraction grating for a laser beam and showed that light diffraction was capable of providing a variety of information about capillary waves.[9] Barter adopted a capacitive antenna for measurement of the dispersion relation of capillary waves on water,[10] Behroozi has directly measured the dispersion relation of capillary waves by laser interferometry.[11] Barik[4] and our previous work[1,5] described light diffraction from liquid surface waves at low frequency. However, there have been few experiments performed to study the dispersion relation of capillary waves by laser diffraction.

In this paper we describe a relatively simple method for the generation of capillary waves of known frequency on a liquid surface in the millimeter wavelength regime. When the laser beam obliquely impinges on capillary waves at a certain angle, steady and visible diffraction spots are formed. A new technique is discussed for measuring their wavelength by light diffraction. The wavelength data provide a very accurate dispersion relation. In particular, based on the dispersion relation, the relation between the surface tension and the temperature is investigated by laser diffraction. The theoretical curves and experimental results are in good agreement. The technique is easy to implement and nondestructive in nature.

II. DESCRIPTION OF EXPERIMENT

A. Experimental setup

A schematic diagram of the apparatus is shown in Fig. 1. A low frequency sinusoidal signal generator is used to produce an output at a few hundred Hertz, which is used to drive the capillary wave exciter. The wave exciter is a delicate triangle metal frame with its vertex connected to an electromagnetic rapping device. The capillary wave exciter produces a quasi-plane wave on the liquid surface. In addition, it is reasonable to assume that the frequency of capillary waves is the same as that of the exciter (driven by a signal generator).[1,3,5] The values of frequency are found directly from the frequency readings of the signal generator. A He-Ne laser beam ($\lambda = 632.8$ nm) is divided by a beam splitter. One of the beams is used to monitor the laser output stability, and the other is directly incident on the liquid surface where capillary waves are traveling. For reference, the diameter of the laser beam cross section is about 3 mm. For an oblique incidence of the laser beam, the shape of the illuminating area on the liquid surface is an ellipse whose major axis and minor axis are about 33 and 3 mm, respectively. The major axis is parallel to the capillary wave direction of travel on the liquid film surface. To increase the diffraction efficiency, the grazing angle θ is chosen to be as small as possible. The liquid pool is filled to a depth of nearly 3 cm and put on a constant temperature regulator. The exciter is installed on a movable holder so that the distance between the wave source and the light incident point can be changed.

Because the screen is fairly far away from the diffraction center (6.76 m in our case), light diffraction from the liquid surface at low frequency is known as Fraunhofer or far-field diffraction. The separation of the diffraction spots becomes greater as the distance between the liquid surface and the screen increases. A CCD (model MTV-1881EX) is used to detect the light distribution. The data detected by the CCD are input into a computer. The CCD size is 7.95×6.45 mm^2 and the signal to noise ratio is better than 48 dB. The diffraction spot can be subsequently displayed, stored, and processed by the computer. The entire experimental setup including the optical system is set on an optical table equipped with pressurized air supports to minimize the possible effect of ambient vibrations.

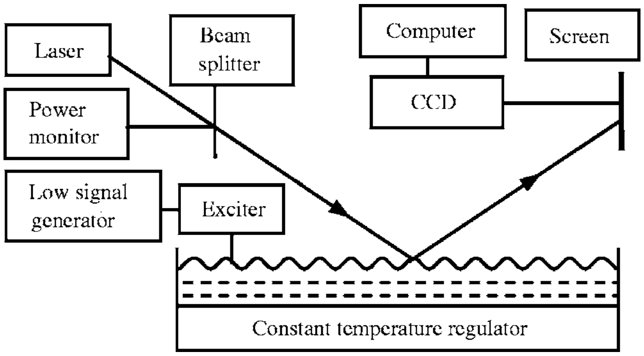

Fig. 1. Schematic diagram of experimental setup.

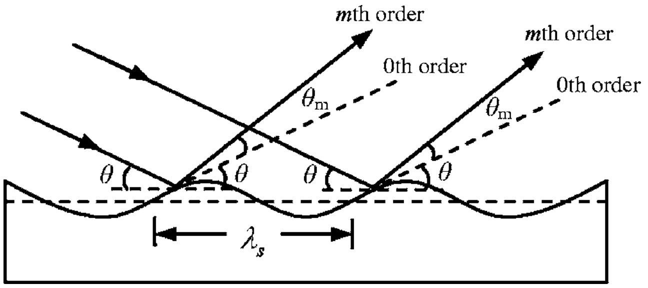

Fig. 3. A reflection liquid surface grating.

B. Theoretical analysis

Although the surface particle motion is somewhat more complex in nature, capillary wave propagation can be adequately approximated as a traveling sinusoidal disturbance and written as

$$y = A\cos(\omega t - kx), \tag{1}$$

where y is the displacement of the surface in the vertical direction, x is a variable along the capillary wave propagation direction, A is the amplitude, ω is the angular frequency, k is the wave number, and $k = 2\pi/\lambda_s$, where λ_s is the wavelength of the capillary waves. As illustrated in Fig. 2, it is assumed that a plane light wave is obliquely incident on the liquid surface, which is disturbed by capillary waves at a few hundred Hertz. The liquid surface acts as a reflection grating for the laser beam (see Figs. 2 and 3), and the Fraunhofer diffraction spots in Fig. 2 are observed on the screen. The wavelength λ_s of capillary waves is the grating spacing. As shown in Figs. 2 and 3, the grating equation for both transmission and reflection is[12,13]

$$\lambda_s[\cos\theta - \cos(\theta + \theta_m)] = m\lambda, \tag{2}$$

where $\theta = 0.09$ is the grazing angle of incidence of the laser beam in our experiment; θ_m is the diffraction angle of the mth-order spot. The integer m represents the order of the various principal maxima. The grating equation (2), for the first-order spot $m = \pm 1$, becomes

$$\lambda_s[\cos\theta - \cos(\theta + \theta_1)] = \lambda \tag{3}$$

and

$$\lambda_s[\cos\theta - \cos(\theta - \theta_1)] = -\lambda. \tag{4}$$

If we use Eqs. (3) and (4), we obtain

$$\lambda_s \sin\theta \sin\theta_1 = \lambda, \tag{5}$$

where θ_1 is the diffraction angle of the first-order spot. The angles θ and θ_1 are calculated by measuring the distances l, d, and h, as shown in Fig. 2. Here $\sin\theta = h/\sqrt{h^2+l^2}$, $\sin\theta_1 = d/(2\sqrt{h^2+l^2})$, h is the perpendicular height of the zeroth-order spot from the liquid surface level, l is the horizontal distance between the location of the laser spot on the liquid surface and the screen, and d is the distance between the positive and the negative first-order diffraction spot on the screen (see Fig. 2). In our experiment, $h = 0.56$ m and $l = 6.76$ m. From Eq. (5) we obtain the expression for the wave number k as

$$k = 2\pi/\lambda_s = \pi h d/\lambda(h^2 + l^2). \tag{6}$$

III. COMPARISON OF THEORY AND EXPERIMENT

The clean air-liquid interface dispersion relation for capillary waves on a deep liquid is given by[14]

$$\omega^2 = \alpha k^3/\rho, \tag{7}$$

where α is the surface tension and ρ is the liquid density. We take the logarithm on both sides of Eq. (7) and obtain

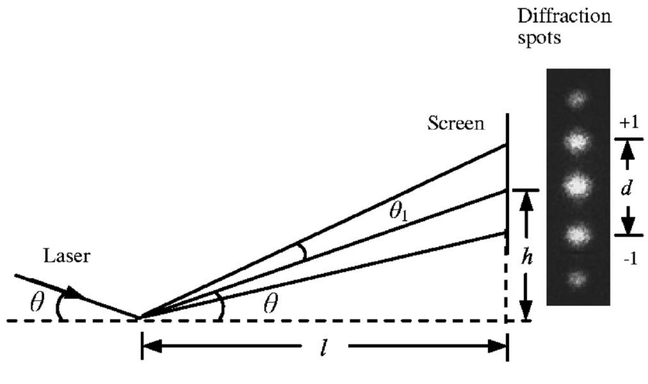

Fig. 2. Light diffraction from capillary waves.

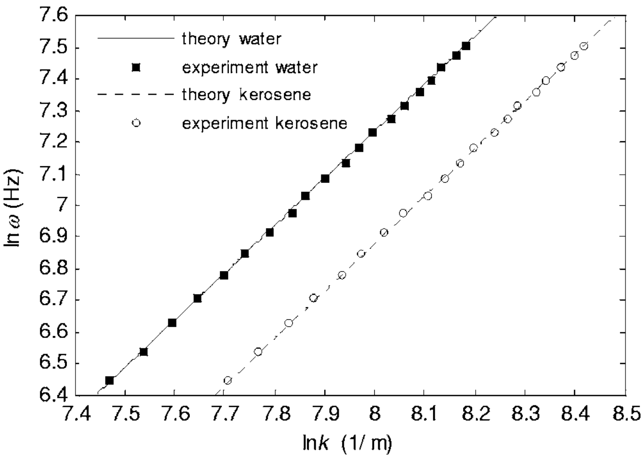

Fig. 4. Log-log plots of the angular frequency ω versus the wave number k on the liquid surface.

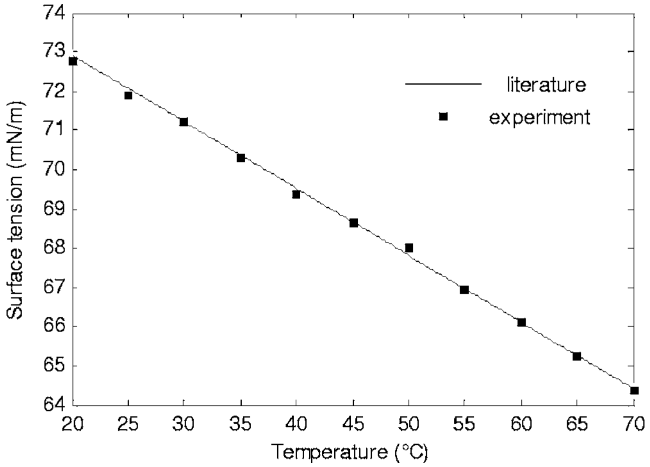

Fig. 5. Temperature dependence of the surface tension of water.

$$\ln \omega = \frac{3}{2} \ln k + \frac{1}{2} \ln\left(\frac{\alpha}{\rho}\right), \quad (8)$$

which is the equation of a straight line for $\ln \omega$ versus $\ln k$ with a slope of $3/2$ and y-intercept $\frac{1}{2}\ln(\alpha/\rho)$.

Sets of diffraction spots are obtained on the screen for different values of the angular frequency ω. According to Eqs. (6) and (7), as ω changes, the distance d and wave number k also change. In our experiment, the CCD captures diffraction spots on the screen, and d is determined by the computer. A careful and accurate measurement of d (of the order of few mm) is crucial for calculating k. To reduce the systematic error in measuring d, we measured d at least three times for each value of ω. The values of d are obtained by averaging these data. Then, using Eq. (6), we calculate the value of k for a particular ω. The dispersion relation for capillary waves can be measured by using the known values of ω and wave number k.

Two kinds of liquid, distilled water and kerosene, were used in the experiment. Capillary waves on their surfaces were generated with the exciter at 20 °C. The frequency of the signal generator is changed from 100 to 290 Hz, and data are gathered at 10 Hz intervals. The CCD captures the corresponding diffraction spots for each set. In Fig. 4 the experimental data are shown by the filled squares (water) and open circles (kerosene), respectively. The solid and dotted lines in Fig. 4 are the theoretical data for water and kerosene, which are obtained from Eq. (8). We used the widely accepted values of the parameters $\alpha = 72.8$ mN/m and $\rho = 998.23$ kg/m^3 for distilled water in Eq. (8) at 20 °C. The accepted values for kerosene are $\alpha = 28.5$ mN/m and $\rho = 800$ kg/m^3 at 20 °C.[15] The theoretical curves match well with the experimental data.

The method can easily be used to measure the surface tension of the liquid (the surface tension appears in the dispersion relation).[15] As discussed in Sec. II, the frequency of the capillary waves increases from 100 to 290 Hz at a certain temperature. Data are gathered at 10 Hz intervals. Using Eq. (6), we calculate the value of k for a particular ω. From this data we make a log-log plot of ω versus k. The slope of the straight line should be consistent with the power law predicted by the dispersion relation [see Eqs. (7) and (8)] and the y-intercept should give the surface tension. If we use the known value of water density ρ for the corresponding temperature,[16] we can calculate the value of the surface tension α. The temperature of the distilled water is changed from 70 °C to 20 °C in 5 °C intervals. Figure 5 displays the experimentally determined surface tension of the distilled water as a function of temperature along with the accepted values.[16] It can be seen that the data agree well with the published values.

Our technique is easy to implement and noncontact in nature. Because surface contamination is a major cause of error in determining the surface tension of a liquid, particularly when covered by a surfactant or monolayer, the method described here provides a distinct advantage.

ACKNOWLEDGMENTS

The authors would like to thank the reviewers for their useful comments. This study was supported by Chinese Educational Ministry under Grant No. 02068.

[a]Electronic mail: 2000zf36392@sina.com

[1] J. Dong, R. C. Miao, J. X. Qi, and F. J. Li, "Unusual distribution of the diffraction patterns from liquid surface waves," J. Appl. Phys. **100**, 033108-1–3 (2006).

[2] F. Behroozi and P. S. Behroozi, "Efficient deconvolution of noisy periodic interference signals," J. Opt. Soc. Am. A **23**(4), 902–905 (2006).

[3] R. C. Miao, X. F. Zhao, and J. Shi, "Modulated interference of reflected light from a liquid surface wave at tens Hertz frequencies," Opt. Commun. **259**(2), 592–597 (2006).

[4] T. K. Barik, A. Roy, and S. Kar, "A simple experiment on diffraction of light by interfering liquid surface waves," Am. J. Phys. **73**(8), 725–729 (2005).

[5] R. C. Miao, Z. L. Yang, J. T. Zhu, and C. Y. Shen, "Visualization of low-frequency liquid surface acoustic waves by means of optical diffraction," Appl. Phys. Lett. **80**(17), 3033–3035 (2002).

[6] F. Behroozi, B. Lambert, and B. Buhrow, "Direct measurement of the attenuation of capillary waves by laser interferometry: Noncontact determination of viscosity," Appl. Phys. Lett. **78**(16), 2399–2401 (2001).

[7] J. D. Barter and P. H. Y. Lee, "Imaging surface-wave analyzer for liquid surfaces," Appl. Opt. **36**(12), 2630–2635 (1997).

[8] W. M. Klipstein, J. S. Radnich, and S. K. Lamoreaux, "Thermally excited liquid surface waves and their study through the quasielastic scattering of light," Am. J. Phys. **64**(6), 758–765 (1996).

[9] G. Weisbuch and F. Garbay, "Light scattering by surface tension waves," Am. J. Phys. **47**(4), 355–356 (1979).

[10] J. D. Barter, "A capacitive antenna for measurement of the dispersion relation of capillary waves on water," Rev. Sci. Instrum. **64**(2), 556–560 (1993).

[11] F. Behroozi and A. Perkins, "Direct measurement of the dispersion relation of capillary waves by laser interferometry," Am. J. Phys. **74**(11), 957–961 (2006).

[12] E. Hecht, *Optics*, 4th ed. (Addison-Wesley, New York, 2002), p. 482.

[13] M. Born and E. Wolf, *Principles of Optics*, 6th ed. (Pergamon, Oxford, 1980), pp. 403–405.

[14] P. G. Klemens, "Dispersion relations for waves on liquid surfaces," Am. J. Phys. **52**(5), 451–452 (1984).

[15] T. M. Bohanon, J. M. Mikrut, B. M. Abraham, and J. B. Ketterson, "Fiber-optic detection system for capillary waves: An apparatus for studying liquid surfaces and spread monolayers," Rev. Sci. Instrum. **62**(12), 2959–2962 (1991).

[16] G. Q. Liu, L. X. Ma, and J. Liu, *Handbook of Chemical Industry and Physics* (Chemical Industry, Beijing, 2002), pp. 3–15.