The Fourier Transformation

Alan Cheville

All the way across the Pacific Sten was bothered by a 28 cycle vibration that seemed to be coming from the left wing. He wished he was sitting on that side of the plane so he could try to see what was causing it. It wasn't dangerous, just annoying. He mentioned it to the pilot after landing, but, as usual, got a look that was somewhere between blank and wary.

"Just tell your maintenance guys to look into it. It probably isn't serious, but you never know" Sten told the captain.

"I wish you wouldn't show off like that. The fact your brain works in the frequency domain makes it nearly impossible for us ordinary time domain people to understand you. Besides it makes us all look like geeks and that pilot was cute. I'm self conscious enough as it is." Elaine, one of the up and coming young scientists in Sten's group, was making her first trip to headquarters.

Sten was about to tell her he wasn't showing off, but before he could get the words out Lenka's elbow in his ribs reminded him he was on vacation. It did feel good to get the family away from the island for a week. But Sten could never leave work behind completely.

Elaine along with Sten and his family checked into the NASA guest house that night, and the next morning the kids spent a day visiting the historical Johnson Space Center and the San Jacinto monument while Sten consulted with his more sound sensitive colleagues at NASA who had moved off-site. Though his old co-workers teased him about being stuck in the middle of the Pacific, it was at best half-hearted. Sten didn't even try to take the bait, the memories were still too painful. The worst moment was the meeting with Joe McDonald, the first project director, and his wife Sheila, team lead on cable aerodynamics. Joe had been one of the first to crack up, and never really got over it. Although the psychiatrists had made it clear that susceptibility was primarily genetic, many who left still felt stigmatized. After the first few incidents most of the engineers were moved off-site. Although the elevator was mainly autonomous, a small contingent immune to the emotional effects of music stayed behind to operate the control station for critical tasks which the speed-of-light latency on the dedicated fiber network centered at Kiribati made it impossible to do in real-time.

The next day, after the meetings, the family drove down the gulf coast to South Padre Island while Elaine stayed behind to network with the NASA brass. Even in the midst of shoppers and beach-side pubs Sten found the quiet unnerving. Both Sten and Lenka were fans of old westerns and the path home to Kiribati took the family first to Goliad and then the Alamo in order to complete the trifecta of Texas independence battle sites. The highway into Goliad was only two lanes with few places to stop and little to see along the way. The kids, who didn't share their parent's enthusiasm for western movies, were

impatient and bored by the time they arrived. Unlike the better known sites, Goliad was not yet tightly controlled so the kids ran free, giving Sten and Lenka time to read the plaques that recreated the massacre at Goliad.

Later that afternoon, the kids were resting in front of the television at the bed and breakfast while Lenka kept an eye on them. Sten took his book and wandered around Goliad, finally stopping under the hanging tree, famed for lynchings of the Mexican cart drivers after Texas won independence. Grooves in the branches gave testimony to the number of deaths the tree had seen. Settling down to read beneath the tree, Sten realized that the part of his brain that continually analyzed the background noise was taking a nap for the first time in years. He had to pay attention to make out the sound of the breeze rustling thousands of live oak leaves overhead. Actually, the sound wasn't as soft as he had first thought, rather it just wasn't noticeable or easy to analyze. Every time he thought he had the spectrum it shifted or changed. Although his intent was to find a quiet place to read, he found he couldn't make himself concentrate well enough to follow the classic text on wavelets he had brought along on vacation to assuage the professional's guilt of even temporarily abandoning his work. Rather he surprised himself by actually relaxing. Having one's back pressed to a tree that had seen hundreds of deaths before his great grandparents were born made it was hard to worry about man's latest experiment gone awry.

When Sten had gotten a job developing software to model dynamic stresses on the space elevator project it has been one of the best moments in his life. The salary allowed him to marry Lenka, and moving to Kiribati after the northern latitudes was idyllic. The area was growing fast as the anchors to the sea floor straddled Jarvis Island, and every atoll seemed to be developing a unique culture as the various countries supplying expertise started communities for the scientists, engineers, and technicians brought in to build the elevator. Local resentment soon faded once the islanders founded a lucrative business selling charcoal from palm fronds and coconut husks for nanotubes.

Sten had spent five happy years figuring out how piezoelectric strain gauges and artificial muscle fibers could be woven into the cable to allow tension to be adjusted dynamically. The time lag of processing data at a central location was too long to let the elevator stay stable under the gravitational and atmospheric forces, so tension was adjusted continually along the cables by autonomous control nodes. Five years went by before the first fiber made contact with earth and the weavers begin the intricate process of layering strands of nanotubes that would ultimately grow into man's first affordable attempt at serious space exploration. Month by month the cable strengthened, and more and more weavers began the long climb to vacuum. In those early days the weavers failed regularly due to exposure to radiation or vacuum. On Friday evenings the engineers would build a fire on the beach, grill shrimp, and watch the fireballs drop into the sea as broken weavers released from the cable and burned up in the atmosphere. One by one the guy cables were attached to the sea floor and the ultimate symmetry of the elevator became apparent; an elliptical gossamer cone rising from the ocean and tapering to a thin line at the limit of vision.

The day the weather changed from still and muggy to the welcome dryness of the trade winds, the music started. The weather branch had accurately predicted the winds picking up, so Sten was with the rest of the dynamic stress control team as the data came in. For the first hour the elevator stayed well within the specifications of no more than .005 radians of deflection over a 1 km section. As the winds picked up, each of the solar powered neural networks adjusted tension in the nanotube cables autonomously, monitored closely by the ground computers running the world's fastest linear predictive coders. However, after the first hour the vibration of the cable would suddenly increase unpredictably over small regions, stimulated by the driving force of the wind. Several seconds later the vibration would be damped by the active systems of the cable, only to pick up again hundreds or thousands of meters away. When Sten finally stepped outside to walk to the cafeteria he heard the music for the first time. The elevator sounded like the world's largest orchestra tuning up. As the hundreds of cable strands oscillated at different frequencies, strange harmonies and overtones were created, only to fade and appear seconds later at a different volume. Like an orchestra preparing for a concert, nearly distinguishable fragments of songs were interspersed with keening, wailings, and progressions of chords.

At first the sound was intoxicating, it never repeated itself exactly. When the vibration moved to the upper atmosphere the music would fade to be almost inaudible, then suddenly swell to an overpowering volume. After a week people started missing the quiet. After a month psychotic episodes began, and families started leaving. The psychiatrists found that people who had perfect pitch, could play instruments, or were otherwise musically talented had their emotional well being affected by the continual sound. They became almost manic-depressive as they responded to subtle tones or barely remembered song fragments. Sound-proofing and noise cancellation didn't help much, since the sound came from hundreds of cables spaced kilometers apart. The idiot, obviously an American, who had the idea of blaring John Phillips Sousa to drown out the sound of the elevator simply made the island into a bastard cross between the sets of *Joe vs. the Volcano* and *A Boy and his Dog*. After months of fruitless work-arounds the conglomerate gave in and paid to move the most affected families off-site.

Three more years had gone by, and they never did manage to stop the music. Over time Sten's team managed to control it some, being able to move a "quiet" zone along the cable to allow safe operation of the elevator car. Some of the graduate students even figured out how to play recognizable music on the cable, although the Halloween attempt at Bach's *Toccata and Fugue in D Minor* set the project back three months when two dozen weavers fell while trying to hit a low D.

[&]quot;Hey buddy, it's getting dark. You have a place to stay?"
"Huh? Uh, yeah, officer. I must have dozed off. I'm staying at the Linburg House Bed and Breakfast."

[&]quot;Nice place. Hey, if you are looking for a good place for dinner, the Steak-N-Stomp has good food and local color."

"Yeah, thanks." Sten stayed under the tree, enjoying the rustling of the leaves, until the fading light made it too dark to read. The kids loved the Steak N Stomp, and stayed up late learning how to line dance. The exercise kept them quiet on the drive to San Antonio and the Alamo, but they were little hellions on the fourteen hour flight back to Kiribati. Sten was never so glad to feel the staccato choppiness of the turbulence created by the cable, and hear the subtle flow of air change over the wing. Several minutes later as the plane descended the music became audible, changing tone through Doppler shift as the plane banked.

"Daddy, what type of whales are those?"

Looking down into the ocean, Sten saw the ring of protective Greenpeace ships riding distant watch on a large pod of whales. The plane was too high to tell which type, but from the size of the pod he guessed humpback. "I don't know, Brigitte. Probably humpback. Why don't you see if you can get the seatback computer to connect to the cameras on the Greenpeace ships so you can see?"

Another of the unintended side effects of the music had been to attract cetaceans from thousands of kilometers. Greenpeace, who initially opposed the elevator, had slowly come around as reefs established themselves on the cable anchors and scientists and activists alike had pushed through a marine life sanctuary in a wide swath around the elevator.

When the plane's doors opened, the music hit Sten. It was vaguely martial today, which helped with getting the kids through the process of picking up bags and loading everyone and everything into the jitney. Since it had been sitting in the sun for almost two weeks, the capacitors were charged enough that they could afford to run the air conditioning the whole way home.

Despite the jet lag, Sten was up before five in the morning the next day and at work an hour early. He wasn't the only one. He'd woken after bad dreams to a monotone that slowly developed beat notes. This happened occasionally as the elevator cables grew in thickness and reached a resonant point. These resonances had become harder to predict as the cable thickness increased; the current resonance hadn't been expected for another week.

"Hey, Sten! You're back. Just in time too, this resonance is not damping so well."

"Yeah, I heard it on the way over. Sounds like there are five sub-harmonics. What time did it start, Felix?"

"The fundamental started rising out of the spectral background yesterday afternoon, but the increase has been a lot faster than we have seen before."

"Has it peaked yet?"

"I knew you were going to ask that. The second derivative just went negative, and we think the peak is going to have an amplitude that tracks the power law you predicted of doubling every three months. The next few are going to be pretty bad. Hey, how is Joe doing?"

"Pretty good. He is really quiet but still on top of things at work."

"Yeah, I'd be quiet too if I beat my wife's face in."

"That isn't fair Felix, it could have been you just as easily."

"I'm sorry, I went to high school with Sheila though, and she didn't deserve that."

The resonance peaked as predicted, and over the next few weeks Sten spent eighteen hour days analyzing the data that had been taken by the cable's embedded sensor network. Although Lenka complained he was not spending enough time with the kids during the school intercession, Sten had his teeth in the problem and couldn't give it up. NASA said both the music and the resonances would die down as the mass of the cable increased. But according to Sten's data not only were the amplitudes increasing, but the frequencies were higher than predicted. This data made no sense at all unless the mass of the cable was less than expected ,or the tension was higher. But all the engineering data said the cable was within specification.

A week later Sten was no closer to an answer, but extrapolating the series over the next five predicted resonance events, it was clear the amplitude of the vibrations would build to a point that could cause damage to the cable. Elaine, who had spent her time in Houston convincing NASA to let her try her solution to this problem, had developed a new algorithm for local stress compensation which they uploaded it along the fiber backbone to each of the distributed processors. Sten didn't harbor much hope that the new code would damp the resonances; in the back of his mind he was afraid the problem came from the driving forces of wind, waves, and tide on the cable. Maybe it was Sten's imagination, but he thought he could hear clusters of creepy minor tones build over the week before the resonance. He felt as if he was an expendable extra in a horror movie, waiting for a killer to jump out of his closet at an unexpected moment.

The stress took its toll. In the twenty four hours before the next resonance event Sten never really got to sleep. Around three in the morning he quit trying. Lenka, getting increasingly used to his erratic behavior woke enough to say, "Try not to wake the kids this time when you leave the house."

The busses weren't running this early, so Sten hiked the back trail through a surprisingly chill, at least for Kiribati, early morning darkness to his lab. In his mind he was already planning a day of tests. Although wrapped up in technical problems, his mind noted the music was subdued, and he could hear tree frogs with an occasional ominous bass rumble. "Tree frogs, that's odd, I thought their population declined as their mating songs became drowned out..." It wasn't until the first orange flash of light he pulled himself

from his own thoughts enough to realize that the rumbles weren't a form of music, rather thunder. Sten ran the rest of the way to the office, but he found the lights already on, and several members of the team tracking the Doppler wind shift of the approaching squall line.

"What is going on? It is dry season and we aren't supposed to have thunder storms."

"Tariq was on watch, Sten, and gave us a call when the storms formed." Elaine said. "They popped up right over Kiribati so we didn't get any warning. Storms are really weird for this time of year, but not unprecedented. The same thing happened in year two, remember, and caused us to redesign the weaver software so they gripped during storms."

"Yeah, and that storm nearly broke the cable and we lost all our weavers in the lower atmosphere. You know a resonance is due any time now, Elaine, Tariq should have called me"

"Sten, you look like crap. You obviously haven't been sleeping well. Hell, none of us have been sleeping well, but you are here all the time. We can handle this. Besides the resonance event started damping around midnight. The stresses are way under what you predicted, and dropping. We would have called if there was a problem developing."

"But that doesn't make sense. I heard the stresses building for a week." Sten peeled the weather map from where it was tacked to the white board and circled Kiribati with his fingers so the display zoomed in. "The winds up there are over 40 knots in some places, gusting to nearly 60." Grabbing a second sheet from his desk, he called up a cross sectional view of the cable and overlaid it on the weather map. Holding both display sheets up to the light he saw Elaine was right. The cable vibration was much less than they had predicted.

Over the next few days Kiribati continued to be buffeted by storms, but the processors within the cable compensated every gust. The weavers, at least the ones at low altitude, stopped work and locked themselves to the cable, so few were lost. The weather continued to be unsettled the rest of the week and the wind and rain thankfully drowned out the music. The team's mood over the week grew more and more elated. A few of the new computer models based on Elaine's code showed the resonances were well below the danger point and decreasing. The only bad news was that the clamping mechanism for the weavers had cut some of the outermost cable strands, and these had frayed in the high storm winds, up to lengths of hundreds of meters.

Sten wasn't convinced. Deep down he knew the next resonance event was going stress the cable nearly to the failure point. As the bad weather cleared off and the trade winds returned, the music remained subdued, almost inaudible. Sten could hear that the local resonances responsible for the music were damped, but why? He reviewed Elaine's code, but it simply ran more efficiently, allowing the piezo-electric strands built into the cable to compensate the local stresses faster. The science hadn't changed, the resonance problem had never been local, Sten would have heard that.

Frustrated with the seemingly insolvable problem of the mysterious resonances, Sten decided he needed time away from the problem for his subconscious to work. Elaine seemed to have day-to-day operations well in hand and she wanted to take responsibility, so Sten turned his attention to the redesign of the weavers. The storm, and resulting damage to the cable, had the Sten working with the engineers responsible for the weavers to solve the long-term problem of cable maintenance. Beyond the problem of redesigning the weavers' clamping mechanism, or claws, so they wouldn't cut cable strands, it wasn't clear how best to repair the damage. One approach was to simply cut away the damaged strands and hope the cable never got dangerously thin at any one point. Another, favored by Sten, was to try to reweave the dangling strands, or at least anchor the ends to the cable to prevent more fraying. In the end both approaches were implemented; each of the redesigned weavers could lay strands, cut them, and splice broken strands together. The engineers, only half jokingly, named the new genus of redesigned weavers 'spiders'. As the engineers desperately worked to ready a swarm of spiders equipped to begin the slow ascent of the cable and fix the dangling strands, the day of resonance got closer.

It became clear the cable would have to survive the next resonance with the cable strands cut during the storm still dangling loose. Some of the strands had frayed further and were nearly a kilometer long. Although the damage to the cable was nowhere near the projected failure point, tension in the cable control room grew in the days approaching the first potentially catastrophic resonance. Sten was the only one who didn't worry. His data showed a clear deviation from the earlier behavior of the cable, something had changed in the forces that drove the resonance. Besides, the music had remained subdued and chaotic even as the strength of the trade winds increased. He would have heard the resonance coming. Elaine, on the other hand, showed the effects of stress. She had a lot riding on the success of her new control code. When Sten remained aloof in the control room, Elaine gradually filled his role as mother hen.

The day of resonance passed with only a slight increase in the local cable tensions. Congratulations poured in from Houston and the other nascent elevator sites around the world. Elaine was promoted to team manager, and Sten moved up to oversee both his old cable monitoring group and the team of engineers responsible for the wavers and spiders. As his first act as an upper level manager, Sten gave the order to release the spiders, who began the slow ascent heavenward cutting or reweaving strands as they went. The weavers continued their work and cable grew thicker. With a seeming disaster averted, the sunny dry days of the trade winds, and praise pouring in from the highest levels of project management, the mood of the whole island became manically positive. Even the music cooperated, building over the days towards a triumphant crescendo.

Sten still worried. Perhaps he had simply developed the habit over years of detail intensive engineering. He kept his doubts to himself though, and ran endless simulations, all of which showed the quicker response due to Elaine's program damping out vibrations, but at the cost of much higher stresses at certain points along the cable length.

As weeks passed and the resonance point approached without any building of vibrations, the island begin to prepare for the penultimate phase of elevator construction; launching the first elevator car to the geostationary anchor point. This would be a dry run of the upcoming "big event", the public opening of the elevator with all the pomp and ceremony of a twenty year, publicly funded project. With the day of the elevator launch approaching, Sten began to hear an edginess to the music. Every tone had accompanying high frequency harmonics so the music sounded as if created by an angry twelve year old, forced to practice each day by her mother, attacking the piano keys.

Sten was at home when disaster finally struck, eighteen hours before resonance. He and Lenka were awoken by a scream that went on and on, rising in volume. Running to the children's rooms he found them awake and unhurt, and his sleep-filled mind finally unraveled that the sound was coming from outside. Lenka was already outside, staring at the sky along with several of their neighbors. Streaks of light from falling spiders radiated outwards from the invisible terminus of the cable, but far fewer than he expected. The clamping mechanisms seemed to be working. "You need to go to work." It was more of a statement from Lenka than a question.

"It will be a long day," said Sten, checking his watch. "at least eighteen and probably twenty four hours."

"Go change, I'll pack you some food. That cafeteria is abysmal."

By the time Sten got to the control room, chaos reigned. By moving from console to console, and having hurried, shouted conversations with individual engineers, Sten pieced together that the piezo-electric actuators along the cable length were beginning to fail, with the result that vibrations were beginning to build to the original and catastrophic levels Sten's original model had predicted. Data was just becoming available on the failure rate, which was still low, but increasing exponentially. Suddenly, above the background of the music, a second higher pitched scream erupted from the front of the room. In the resulting silence, as conversations were cut off in mid word, all eyes turned to Elaine.

"Sorry about that, but I had to get your attention. Listen people, we have to work fast. There are less than six hours before the cable failure probability rises above ten percent, and about twelve hours before failure is statistically certain. We know what is going on. The response time of the actuators has been decreased over the past weeks due to our improvements in code so it is much faster than the response time of the sensors. This means that we are getting transient stresses strong enough to push the actuators to failure that we can only measure indirectly. The data on cable stresses has been wrong for weeks, but we didn't know it until we hit the failure point an hour ago. There is not much we can do about it now. The best option to get this thing under control is to reprogram the actuators, effectively slowing them down. This will keep the actuators from failing so we can maintain control over the cable, but puts us back in the situation where we have a potentially catastrophic resonance to deal with.

"If you aren't directly involved with the following teams..." Elaine rattled off a list of technical groups, "...you are welcome to stay and contribute in any way possible. We need all the ideas we can get. But shut up and stay out of the way. I want to see the group leaders up here now!"

As Sten moved to the front of the room to listen in on the group leaders meeting, notice the sound of the room had subtly changed. The level was just as high, but it was easier to hear individual conversations. Words were more purposeful and focused, the edge of hysteria had been taken off and information traveled farther. Elaine's announcement had helped restore much needed order to the control room.

Sten sat in on the group leaders meeting until it broke up, then wandered around the room listening in on conversations. Eventually he ended up back at the weaver control station. Sten had little to contribute since his weavers were gripping the cable for dear life. He was heartened to see that very few had fallen after the initial vibrations, but their long term survival depended on the cable holding, which looked increasingly less likely. As the hours wore on and actuator speeds were slowed to match those of the sensors, the cable vibration returned to the pattern originally predicted by Sten. The driving forces of wind and tide, and the slower variations in gravitational pull of the moon and sun, were causing the space elevator to vibrate like a plucked string.

The conversation in the room became increasingly chaotic and hysterical, and Sten had to strain to hear what was being said at even the closest work stations. Eventually he gave up, and leaned back in his chair, imagining life after the fall of the cable. They would move again, somewhere. It would be nice to spend more time with Lenka and the kids, perhaps spend every afternoon under a big oak tree reading, like he had in that place in Texas. Where was it... Goliad. Funny how people were like leaves. You get enough of them contributing random noise and you can't make out individuals anymore. All the sound they have to contribute blends together. It was only by making them work together, creating coherent motion, that something like the elevator could be built. And even then chaos would eventually win. But this wasn't chaos, this was a harmonic vibration. A single frequency, collective motion of God's harp string. Suddenly Sten knew how to save the cable and knew no-one else in the room would hear the solution over the music.

Slowly, almost languidly, he turned to his console and sent a command to every weaver to cut into the cable. All along the cable length—at whatever random positions weavers held their death-grip on the cable—small nitride tipped blades whirred in to action, cutting shallow grooves through the strands of carbon nanotubes. Families looking skyward as the dawn came slowly to Kiribati saw the cable slowly become surrounded by a glowing, growing halo. In the atmosphere the carbon fibers begin to slowly unwind. Ten meters, one hundred meters, the strands whipped in the trade winds, adding their own chaotic, unpredictable forces to the vibration. Actuators responded to these new forces as well as the collective forces which drove the resonance. Slowly the new forces broke the coherent oscillation of the cable and the vibration started to dampen. Across the room consoles reflected the new data and new forces, and the sound of conversation

doubled, and doubled again and engineers sought to make sense of what was going on. Sten quietly slipped outside.

The scream was diminishing, fading back to music, which—over the course of the next hour—became white noise.

Sten's remaining family surprised him for his eightieth birthday by throwing a party at the Yggdrasil Lounge, named from the world tree in Norse mythology. The lounge was in the poshest hotel at Thousand Kilometer Waypoint, built at the transition point high above Kiribati. It was really too bad Lenka wasn't around any more, she would have loved the view and seeing how the island had changed since the years they had spent there.

It had taken nearly a day to make the trip from the trunk, where roots split off to run deep down into the earth's crust tapping geothermal power, up to past the countless carbon branches radiating out from the trunk to the twigs and solar collecting leaves. The tram car had even passed one of the rare and expensive AI beings, an Emancipated Neural-network Tree Synthesizers shepherding a band of Sten's own symbiotic programmable inter-dependently educatable re-weavers busy making repairs to a branch. The spiders had evolved with the tree.

Some of Sten's surviving colleagues had made the trip down from the higher colonies where they had retired after years of gravity began to take its toll. They kept telling him to move, how wonderful life was above the atmosphere. But Sten thought he would have trouble leaving growing things, weather, and the chaos of Earth behind. Thousand Kilometer Waypoint, the transition point, marked the boundary between the tree and the cable, and the Yggdrasil Lounge had a good view in both directions. Down towards the earth Sten's view was obscured by the tree itself. The chaotic tangle of branches shone in the sun, with solar collectors sending back flickering reflections as they vibrated with the cable or more drastic local movements caused by ents or spiders.

Looking straight outwards, the curve of the earth was clearly visible, with the atmosphere a thin shell fading into the black of space. It was a clear day. Samoa was a green discontinuity in the Pacific off to the southwest, and he could just make out Fiji on the horizon (I fricking love Google Earth!).

Pulling his gaze away from Fiji and looking up, and then up again, Pisces was visible uncounted kilometers distant, and above that the stars that formed Ares. Directly above him, the cable stretched to the bright point of light that was the geosynchronous terminus. Along the cable, fading out of sight like a string of pearls, were the seemingly fragile bubbles of the low gravity colonies. The colonies counterbalanced the increasing mass of the geosynchronous terminus as more and more industry moved heavenward. Things seemed clean and simple up here, unlike the hassles and chaos of terrestrial life. Plus it was quiet, really quiet. It let a man think. And you could see the stars, almost impossible

from Earth. Funny, the stars seemed randomly thrown down, like sugar on black velvet, but if you looked long enough patterns emerged. Chaos was necessary for life on earth, but things changed up here on the edge of space. The rules were different, simpler. Maybe it was time to move upwards.